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and Public-Sector Pricing:
A Contribution to the Foundations of Public Economics

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Abstract

The paper develops an integrated model of optimal nonlinear income taxation, public-goods provision and pricing in a large economy. With asymmetric information about labour productivities and public-goods preferences, the multidimensional mechanism design problem becomes tractable by requiring renegotiation proofness of the final allocation of private goods and admission tickets for excludable public goods. Under an affiliation assumption on the underlying distribution, optimal income taxation, public-goods provision and admission fees have the same qualitative properties as in unidimensional models. These properties are obtained for utilitarian welfare maximization and for a Ramsey-Boiteux formulation with interim participation constraints.

Key Words: Optimal Income Taxation, Public Goods, Public-Sector Pricing, Multidimensional Mechanism Design, Ramsey-Boiteux Pricing

JEL Classification: D 82, H 20, H 40

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1 Introduction

This paper develops an integrated model of optimal nonlinear income taxation and public-goods provision and pricing in a large economy. The model is useful for determining under what conditions and in what sense the availability of income taxation, as a source of government finance, alleviates the tension between incentive constraints and efficiency, redistribution concerns and financing needs in the provision of public goods. I am particularly interested in the respective roles of income taxes and of admission fees on excludable public goods.

The paper shows that the traditional alignment of optimal nonlinear income taxation with utilitarian concerns for distribution and of public-goods provision and pricing with government budget constraints and finance is inappropriate. Nonlinear income taxation is important for covering government financing needs, and the pricing of public goods can be important for utilitarian redistribution. For taxes and public-sector prices, the appropriate dividing line is not between instruments used for redistribution and instruments used for finance, but between instruments that are vulnerable to arbitrage through side-trading among participants and instruments that are not vulnerable to such arbitrage.

The analysis takes a step towards integrating the three subfields into which normative public economics has traditionally been divided: the theory of public-goods provision; the theory of indirect taxation and public-sector pricing; and the theory of optimal income taxation. Until recently, these subfields have mostly been studied separately and relations between them have been little explored: The theory of public-goods provision focusses on the elicitation of preferences for public as opposed to private goods; the theory of indirect taxation and public-sector pricing à la Ramsey-Boiteux is concerned with the combination of indirect taxes and public-sector prices used to finance a given public-sector spending requirement when lump sum taxation is unavailable; the theory of optimal income taxation studies the equity-efficiency tradeoff that arises when earning ability is unobservable.

Except for the seminal work of Atkinson and Stiglitz (1976), areas of overlap – and possible conflict! – between these different lines of investigation have received little attention. Distributive concerns play only a secondary role in the literature on public-goods provision,¹ indirect taxation and public-sector pricing.² Elicitation of preferences for public goods plays

¹Exceptions are Christiansen (1981), Boadway and Keen (1993), Blomquist and Christiansen (2001), Bierbrauer (2002), Cremer and Laffont (2003), Gaube (2003).

²Diamond and Mirrlees (1971) show that the simple elasticities rule of the Ramsey-

hardly any role in the literatures on direct and indirect taxation.³ Most importantly, the treatment of government budget constraints, financing needs and cross-subsidization between activities has not been unified.

As an example, consider the constraints that incentive considerations impose on the relation between the elicitation of public-goods preferences and people's financial contributions. The mechanism design approach to public-goods provision has shown that these constraints induce a conflict between first-best efficiency, feasibility and individual rationality so that, in some settings, the need to finance the public good from voluntary contributions precludes the attainment of an efficient allocation.⁴ Thus in a large economy with independent private values, it is impossible to have positive levels of public-goods provision financed by voluntary contributions: Because, in such an economy, the cross-section distribution of preferences is independent of any one person's preferences, nobody is ever pivotal for determining public-goods provision levels, and therefore nobody is willing to make a voluntary financial contribution towards financing them.

For *excludable* public goods, Schmitz (1997) and Norman (2004) have suggested that the costs of public-goods provision be covered by admission fees. In a large economy with independent private values, the inefficiency of not having the public goods at all would thus be avoided. However, there would be an – admittedly smaller – inefficiency from excluding people with low positive valuations even though, with nonrivalry in consumption, the marginal cost of admitting them to the enjoyment of the public goods would be zero. The question arises why the excludable public goods should be financed from distortionary admission fees rather than any other sources of funds available to the government.

The question is closely related to the Atkinson-Stiglitz (1976) critique of the Ramsey-Boiteux approach to indirect taxation and public-sector pricing under a government budget constraint. According to that critique, the very question addressed in the Ramsey-Boiteux approach is moot if direct taxation is available to cover the government's financing needs. In an economy with *homogeneous* consumers, there is no reason to rule out lump-sum

Boiteux approach must be replaced by a weighted-elasticities rule when distributive concerns play a role. Cremer et al. (2001, 2003) and Golosov et al. (2003) discuss the use of indirect taxes for distributive purposes when private-good endowments or savings from past periods as well as current labour productivities differ across people. Cremer and Laffont (2003) consider the implications of distributive concerns for public-sector pricing when people with different incomes also have different costs of accessing the public good or service.

³The exception is Bierbrauer (2002).

⁴Güth and Hellwig (1986), Rob (1989), Mailath and Postlewaite (1990).

taxation as a source of government finance, and a deviation of consumer prices from marginal costs is undesirable. In an economy with *heterogeneous* consumers having different earning abilities, income taxation suffices for optimal utilitarian redistribution and government finance. Therefore a deviation of relative consumer prices from relative marginal costs is again undesirable, *unless* nonseparabilities in utility functions generate patterns of complementarity between consumption goods and leisure so that distortionary indirect taxes and public-sector prices can be used to improve the equity-efficiency tradeoff which arises from the unobservability of individual earning abilities.⁵

The use of admission fees to finance the provision of excludable public goods is akin to the use of indirect taxes and public-service prices exceeding marginal costs in the Ramsey-Boiteux approach. In principle, therefore, the Atkinson-Stiglitz critique applies to the Schmitz-Norman model of excludable public goods as well as to the Ramsey-Boiteux theory of indirect taxation and public-sector pricing. In both cases, a specific financing requirement is subject to the criticism that additional finance would be available from direct taxation.

However, there is an important difference. Whereas Atkinson and Stiglitz assume that people differ only with respect to their earning abilities, in the public-good provision problem, they also differ with respect to their preferences for public versus private goods.⁶ This difference gives rise to two objections to the Atkinson-Stiglitz recommendation. First, the use of direct taxes for public-good finance may violate interim individual-rationality constraints. Thus in the large economy with independent private values, the Atkinson-Stiglitz critique calls for first-best levels of public-goods provision, financed by equal lump-sum payments from all participants. Participants who do not care for the public goods at all consider this arrangement to be worse than one where no public goods are provided and no taxes are levied. If one has no qualms about the unrestricted use of the government's power of coercion, one may not be bothered by this finding. It does, however, require abandoning the voluntary-exchange approach to public economics that underlay Lindahl's (1919) creation of the theory of public goods.⁷ If, instead,

⁵Thus, Cremer et al. (2003) and Golosov et al. (2003) find that capital income taxation can be useful for enhancing the scope for redistribution through labour income taxation.

⁶Cremer et al. (2001) have pointed out that Atkinson and Stiglitz rely very heavily on earning abilities being the *only* source of heterogeneity. Allowing for heterogeneity in private-goods endowments, they obtain a rationale for redistributive indirect as well as direct taxation.

⁷The notion that relations between the individual and the state involve a quid pro quo

one imposes a participation constraint, one is led right back to the analysis of Schmitz (1997) and Norman (2004) where feasibility requirements and participation constraints combine to give rise to a government budget constraint à la Ramsey-Boiteux. Indeed, for a model with multiple excludable public goods, Hellwig (2004 a) shows that, if public goods cannot be bundled, the problem of designing an optimal mechanism for public-goods provision subject to feasibility and individual rationality constraints is exactly equivalent to the Ramsey-Boiteux problem for setting optimal admission fees.⁸

Second, even if one is not worried about participation constraints, the heterogeneity in public-goods preferences must be taken into account in utilitarian welfare maximization. Because people with different public-goods preferences achieve different payoffs, this heterogeneity itself is a source of distributive concerns. Thus in the Schmitz-Norman model of the provision of an excludable public good, the people who do not care for the public good at all have the lowest payoffs. The Atkinson-Stiglitz recommendation to finance the public good by a lump-sum tax rather than by admission fees would reduce these people's payoffs even further while benefiting those who get a lot of enjoyment out of the public good and are therefore already better off. If a utilitarian planner is inequality averse, concern about this distributive effect can justify the use of admission fees rather than lump-sum taxes. As discussed in Hellwig (2003), the admission fees for excludable public goods then serve to appropriate some of the benefits of people who enjoy the public good a lot and to increase the private-good consumption of people who don't benefit from the public good at all. In the Rawlsian limit of infinite inequality aversion, it is actually desirable to set admission fees at monopoly levels so as to maximize the addition to private-good consumption of the worst-off people in the economy.

The present paper takes this analysis one step further by integrating the analysis of public-goods provision with the theory of income taxation. In a model involving heterogeneity in earning abilities as well as public-goods preferences, I will study the use of income taxes for both redistribution and

is much older, going back to Grotius and Locke. For an extensive account of the history of this notion, see Musgrave (1959), pp. 61 - 89. In the present context one may doubt whether it is appropriate to refer to *interim* individual rationality rather than *ex ante* individual rationality as the concept of voluntariness. *Ex ante* individual rationality is problematic if the "real" situation is considered to be one of incomplete information, and the *ex ante* stage and common prior are only modelling devices used to make the problem with incomplete information tractable by turning it into one with imperfect information.

⁸However, the allocation induced by an optimal mechanism with bundling may be superior to the optimal Ramsey-Boiteux allocation; see Fang and Norman (2003), Hellwig (2004 a).

public-goods finance, side by side with admission fees for excludable public goods. Whereas Hellwig (2003) had replaced heterogeneity in earning ability as a source of distributive concerns by heterogeneity in public-good preferences, I now look at both dimensions of heterogeneity jointly, developing a formalism to investigate their respective implications for the use of income taxes and public-goods admission fees as redistribution devices. I also study the joint use of income taxation and public-goods admission fees as sources of public-goods finance when lump-sum taxes are restricted by participation constraints.

Optimal income taxation and the provision and pricing of public goods are treated as a matter of Bayesian mechanism design. By a large-numbers effect, the cross-section distribution of earning abilities and public-goods preferences is taken to be given and commonly known. However, each person's individual characteristics are private information of that particular person. With heterogeneity in earning abilities and in tastes for m public goods, there are $m + 1$ hidden characteristics, so the information asymmetry constraining the choice of an allocation or mechanism is inherently multidimensional.

Because of the well-known difficulties of mechanism design with multidimensional incentive constraints, I am unable to solve the multidimensional mechanism design problem in full generality. Following an approach pioneered by Hammond (1979, 1987) and Guesnerie (1995), I restrict the analysis by imposing an additional requirement of *renegotiation proofness*. Under this requirement, the final allocation of private goods and of admission tickets for excludable public goods must not provide participants with any incentive to engage in side trading out of sight of the mechanism designer. The mechanism designer is assumed to have no control over such side trading. If the allocation that he stipulates were to leave room for people to engage in (incentive compatible) Pareto improving trades with each other, they would use this opportunity and trade to a final allocation where there would be no further room for such trades. Given that the final allocation after side trading must be renegotiation proof, the mechanism designer who cares only about final outcomes may as well restrict his initial choice to renegotiation proof allocations.

In a large economy, the renegotiation proofness condition is satisfied if and only if the final allocation of private goods and of admission tickets for public goods is Walrasian, i.e. supported by a price vector.⁹ For the

⁹In Hellwig (2004 a) this observation is used to show that renegotiation proofness eliminates the possibility of bundling as well as randomization.

usual quasi-linear specification of public-goods preferences, it follows that final holdings of admission tickets for a public good depend *only* on the values of the preference parameter for that public good; in particular, they are independent of earning abilities. In combination with incentive compatibility, this independence property in turn implies that a person's labour supply and output provision depend *only* on the person's earning ability and are independent of that person's public-goods preferences. Given the latter independence property, incentive constraints for labour supply and output provision take the same form as in the standard optimal-income-tax problem. The imposition of renegotiation proofness thus reduces the $m + 1$ -dimensional mechanism design problem to a problem involving $m + 1$ *unidimensional* incentive constraints, one for each public good and one for labour supply and output provision. This problem can be handled by standard methods.

For the second-best utilitarian welfare maximization problem, the analysis shows that, under an affiliation assumption on the distribution of the hidden characteristics, optimal income taxes and optimal admission fees exhibit the same features as in the corresponding unidimensional problems. In particular, under positive inequality aversion of the mechanism designer, the optimal marginal income tax rate is positive in the interior of the domain of relevant income levels and zero at the boundaries; optimal admission fees are zero if inequality aversion is small; they are close to profit-maximizing monopoly prices if inequality aversion is large. The affiliation assumption admits independence as well as positive correlations, but rules out negative correlations of the hidden characteristics. Negative correlations might lead to a situation where people with large earning-ability realizations get so little pleasure from public goods that the social marginal utility of private-good consumption of these people is very high and they should be on the receiving end of private-good redistribution through income taxation. This possibility is eliminated by the affiliation assumption.

The structure of the conditions characterizing optimal income taxes and admission fees does not change if, in addition to feasibility and renegotiation proofness, an interim participation constraint is imposed. In terms of the formalism, the imposition of a participation constraint affects only the Lagrange multiplier of the feasibility condition. This Lagrange multiplier goes up and the feasibility constraint is seen to be more stringent because, when the participation constraint is binding, the implications of, e.g., a reduction in an admission fee cannot be neutralized by a lump-sum reduction in private-good consumption, which would be nondistortionary, but must be neutralized by a distortionary increase in income taxes or in another ad-

mission fee. However, this is the only change induced by the imposition of interim individual rationality.

In the absence of inequality aversion, the mechanism design problem with interim individual rationality is equivalent to a generalized version of the Ramsey-Boiteux problem. In this generalized version, revenues from nonlinear income taxes, as well as admission fees, are used for public-goods finance. If participation constraints are binding, the optimal marginal income tax is positive even at the bottom of the income distribution. Otherwise the formalism for the optimal nonlinear income tax is the same in the Generalized Ramsey-Boiteux Problem as in the utilitarian approach of Mirrlees (1971, 1976) and Seade (1977, 1982) or in the utilitarian model without participation constraints.

Optimal admission fees satisfy a version of the inverse-elasticities rule; if the mechanism designer is inequality averse, the inverse-elasticities rule is replaced by a weighted-inverse-elasticities rule à la Diamond-Mirrlees (1971). The form of this rule is again independent of whether participation constraints are imposed or not. Similarly, the difference between utilitarian welfare maximization and the Generalized Ramsey Boiteux Problem, is *not* relevant for the validity of the Mirrlees-Seade characterization of optimal income taxes and of the weighted-inverse-elasticities characterization of optimal admission fees.

In the following, Section 2 lays out the basic model, explaining the requirements of incentive compatibility, renegotiation proofness and feasibility and showing how renegotiation proofness and incentive compatibility together turn the $m + 1$ -dimensional mechanism design problem into one that has $m + 1$ unidimensional incentive constraints.

Section 3 studies optimal allocations. Section 3.1 characterizes first-best, i.e., optimal feasible allocations. Section 3.2 characterizes second-best allocations, i.e., optimal incentive compatible, renegotiation proof and feasible allocations when the mechanism designer is inequality averse. The implications of imposing interim individual rationality are considered in Section 3.3. All proofs are given in the appendix.

2 A Model with Multiple Public Goods and Heterogeneous Labour

2.1 The Basic Model

I study a large economy with one private good, m public goods and labour. Some of the public goods are excludable, some may be nonexcludable. The sets of excludable and nonexcludable public goods are denoted as J^e and J^{ne} . For each individual h in the economy, an allocation must determine how much of the private good the individual gets to consume, which public goods he is admitted to and how much labour input he provides. Let Q_1, \dots, Q_m be the levels at which public goods $1, \dots, m$ are provided. Individual h with taste parameters θ_i^h , $i = 1, \dots, m$, obtains the utility

$$c^h + \sum_{i \in J^h} \theta_i^h Q_i - \ell^h. \quad (2.1)$$

if he has private-good consumption c^h , if he is admitted to the enjoyment of public goods $i \in J^h$, and if he provides the labour input ℓ^h .

The labour input ℓ^h serves to produce an output $y^h = \varphi(\ell^h, n^h)$, where n^h is a productivity parameter pertaining to h . It is actually more convenient to think of ℓ^h and y^h in terms of the output y^h and the associated input requirement

$$\ell^h = \gamma(y^h, n^h), \quad (2.2)$$

where, $\gamma(\cdot, n^h)$ is the inverse of the production function $\varphi(\cdot, n^h)$. In this notation, participant h with taste parameters θ_i^h , $i = 1, \dots, m$, and productivity parameter n^h obtains the utility

$$c^h + \sum_{i \in J^h} \theta_i^h Q_i - \gamma(y^h, n^h) \quad (2.3)$$

if he has private-good consumption c^h , if he is admitted to the enjoyment of public goods $i \in J^h$, and if he provides the output y^h . The function γ is assumed to be twice continuously differentiable, increasing and strictly convex in y^h as well as nonincreasing in n^h . Moreover, $\gamma(0, n^h) = 0$, $\lim_{y^h \downarrow 0} \gamma_y(y^h, n^h) = 0$ for all n^h , and $\gamma_{yn}(y^h, n^h) < 0$ for all y^h and n^h .

The productivity parameter n^h and the vector $\theta^h = (\theta_1^h, \dots, \theta_m^h)$ of preference parameters are the realizations of random variables \tilde{n}^h and $\tilde{\theta}^h$ taking values in $[0, 1]$ and $[0, 1]^m$, which are defined on some underlying probability space (Ω, \mathcal{F}, P) . The joint distribution $F(\cdot)$ of \tilde{n}^h and $\tilde{\theta}^h$ is assumed to

be the same for all agents. Moreover, F has a strictly positive, continuously differentiable density $f(\cdot)$. The marginal distributions of \tilde{n}^h and $\tilde{\theta}_i^h$, $i = 1, \dots, m$, are denoted as F^n and F^i , their densities as f^n and f^i .

The set of participants is modelled as an atomless measure space (H, \mathcal{H}, η) . I assume that the random variables $(\tilde{n}^h, \tilde{\theta}^h)$, $h \in H$, are independent and that, by a large-numbers effect, with probability one, the probability distribution F can be taken to be the cross-section distribution of the pair $(\tilde{n}^h(\omega), \tilde{\theta}^h(\omega))$ in the population. Thus, for almost every $\omega \in \Omega$, I write

$$\frac{1}{\eta(H)} \int_H \varphi(\tilde{n}^h(\omega), \tilde{\theta}^h(\omega)) d\eta(h) = \int_{[0,1]^{m+1}} \varphi(n, \theta) dF(n, \theta) \quad (2.4)$$

for every F -integrable function φ from $[0, 1]^{m+1}$ into \mathbb{R} .¹⁰

I restrict the analysis to allocations that satisfy an *ex-ante neutrality* or *anonymity* condition. The level c^h of an individual's private-good consumption, the set J^h of public goods to which the individual is admitted, and the level y^h of output that the individual provides are assumed to depend on h and on the state of the world x *only* through the realizations $\tilde{n}^h(\omega) = n^h$ and $\tilde{\theta}^h(\omega) = \theta^h$ of the random variables \tilde{n}^h and $\tilde{\theta}^h$. In principle, c^h , J^h and y^h should also depend on the cross-section distribution of the other agents' parameter realizations $\tilde{n}^{h'}(\omega) = n^{h'}$ and $\tilde{\theta}^{h'}(\omega) = \theta^{h'}$ in the population, but because this cross-section distribution is constant and independent of ω , there is no need to make this dependence explicit. This is a major advantage of working with the large-economy specification with the law of large numbers.

An *allocation* is thus defined as an array

$$A = (\mathbf{Q}^A, c^A(\cdot, \cdot), y^A(\cdot, \cdot), \chi_1^A(\cdot, \cdot), \dots, \chi_m^A(\cdot, \cdot)), \quad (2.5)$$

such that $\mathbf{Q}^A = (Q_1^A, \dots, Q_m^A)$ is a vector of public-good provision levels, and $c^A(\cdot, \cdot), y^A(\cdot, \cdot), \chi_1^A(\cdot, \cdot), \dots, \chi_m^A(\cdot, \cdot)$ are functions which stipulate for each $(n, \theta) \in [0, 1]^{m+1}$, a level $c^A(n, \theta)$ of private-good consumption, a level $y^A(n, \theta)$ of output provision and indicators $\chi_i^A(n, \theta)$ for admission to public goods $i = 1, \dots, m$, to be applied to any participant h in the state ω if $(\tilde{n}^h(\omega), \tilde{\theta}^h(\omega)) = (n, \theta)$. The indicator $\chi_i^A(n, \theta)$ takes value one if the consumer is admitted and value zero, if he is not admitted to public good i .

¹⁰As discussed by Judd (1985), the law-of-large-numbers property (2.4) is consistent with, though not implied by stochastic independence of the random pairs $(\tilde{n}^h, \tilde{\theta}^h)$, $h \in H$. For a large-economy specification with independence in which the law of large numbers holds as a theorem, see Al-Najjar (2004).

Through (2.2), the output level $y^A(n, \theta)$ also determines a labour input level $\ell^A(n, \theta) = \gamma(y^A(n, \theta), n)$.

Allocations are assessed according to the cross-section distribution of utility which they induce, according to the utilitarian welfare functional

$$\frac{1}{\eta(H)} \int_H W \left(c^A(\tilde{n}^h, \tilde{\theta}^h) + \sum_{i=1}^m \chi_i^A(\tilde{n}^h, \tilde{\theta}^h) \tilde{\theta}_i^h Q_i^A - \gamma(y^A(\tilde{n}^h, \tilde{\theta}^h), \tilde{n}^h) \right) d\eta(h),$$

which under (2.4) is almost surely equal to

$$\int_{[0,1]^{m+1}} W \left(c^A(n, \theta) + \sum_{i=1}^m \chi_i^A(n, \theta) \theta_i Q_i^A - \gamma(y^A(n, \theta), n) \right) dF(n, \theta). \quad (2.6)$$

The function $W(\cdot)$ is assumed to be twice continuously differentiable, strictly increasing, and concave. If $W(\cdot)$ is affine, i.e. if $W''(v) = 0$ for all v , the welfare functional (2.6) is ordinally equivalent to the aggregate surplus

$$\int_{[0,1]^{m+1}} [c^A(n, \theta) + \sum_{i=1}^m \chi_i^A(n, \theta) \theta_i Q_i^A - \gamma(y^A(n, \theta), n)] dF(n, \theta). \quad (2.7)$$

In this case, the cross-section dispersion of utility levels is of no concern. In contrast, if $W(\cdot)$ is strictly concave, the mechanism designer is *inequality averse* and would like to use the assignment of private-good consumption to reduce the cross-section dispersion of utility levels. Following Atkinson (1973), I refer to the relative curvature $\rho_W(v) = -\frac{W''(v)}{W'(v)}$ as a *measure of inequality aversion*.

The mechanism design problem will be to choose an allocation that maximizes the welfare functional (2.6) over the set of *admissible* allocations. Admissibility will be defined with reference to *incentive compatibility*, *renegotiation proofness*, *feasibility* and, in the last part of the paper, *individual rationality*. In the following, I explain these requirements.

2.2 Incentive Compatibility

Each participant $h \in H$ knows the realizations n^h and θ^h of his own productivity and preference parameters, and hence the payoff

$$v^A(n^h, \theta^h) := c^A(n^h, \theta^h) + \sum_{i=1}^m \chi_i^A(n^h, \theta^h) \theta_i^h Q_i^A - \gamma(y^A(n^h, \theta^h), n^h) \quad (2.8)$$

that he obtains from an allocation A .

The information about n^h and θ^h is private. Apart from the distribution F , nobody knows anything about the pair $(\tilde{n}, \tilde{\theta})$ pertaining to somebody else. Following Mirrlees (1971) and the subsequent literature, I also assume that labour inputs are unobservable. Therefore there is nothing to prevent a participant with productivity and preference parameters n, θ from claiming to have parameters n', θ' in order to obtain the private-good consumption $c^A(n', \theta')$ as well as public-goods admissions according to the indicators $\chi_i^A(n', \theta')$ while producing the output $y(n', \theta')$ with the labour input $\gamma(y(n', \theta'), n)$. An allocation A is said to be *incentive compatible* if the participant has nothing to gain from such a claim, i.e. if and only if

$$v^A(n, \theta) \geq c^A(n', \theta') + \sum_{i=1}^m \chi_i^A(n', \theta') \theta_i Q_i^A - \gamma(y^A(n', \theta'), n) \quad (2.9)$$

for all (n, θ) and (n', θ') in $[0, 1]^{m+1}$.

A characterization of incentive compatible allocations is beyond the scope of this paper. As discussed by Rochet and Choné (1998), the fact that the incentive problem is multidimensional and that the payoff function (2.8) is not affine in n makes it all but impossible to find a simple characterization.

2.3 Renegotiation Proofness

In addition to incentive compatibility, I impose a condition of *renegotiation proofness*. The mechanism designer is assumed to be unable to verify the identities of people who present tickets for being admitted to the enjoyment of a public good. In particular, he is unable to check whether the people who present tickets for admission to a public good are in fact the same people to whom the tickets have been issued. As in Hammond (1979, 1987) and Guesnerie (1995), he is also unable to prevent people from trading admission tickets and the private good among each other. If the initial allocation of tickets leaves room for a Pareto improvement through such trading, then, as discussed by Hammond and Guesnerie, in the absence of transactions costs, such trading will occur, and the initial allocation will not actually be the final allocation.

Imposition of renegotiation proofness in this setting corresponds to the idea that, regardless of the allocation that is initially chosen by the mechanism designer, in the absence of transactions costs, any allocation that is finally implemented must itself be renegotiation proof. If the mechanism designer is aware of the possibility of renegotiation and if he cares about the

allocation that is finally implemented rather than the one that is initially chosen, his choice may be directly expressed in terms of the final renegotiation proof allocation. Indeed if he chooses a renegotiation proof allocation from the beginning, this initial allocation will also be the final allocation.

For a formal treatment, I introduce the concept of a *net-trade allocation* for private-good consumption and public-good admission tickets as an array $(z_c(\cdot, \cdot), z_1(\cdot, \cdot), \dots, z_m(\cdot, \cdot))$ such that for each (n, θ) , $z_c(n, \theta)$ and $z_1(n, \theta), \dots, z_m(n, \theta)$ are the net additions to private-good consumption and admission ticket holdings for public goods of a consumer with productivity parameter n and preference parameter vector θ . Given an initial allocation A , a net-trade allocation $(z_c(\cdot, \cdot), z_1(\cdot, \cdot), \dots, z_m(\cdot, \cdot))$ is *feasible* if $\chi_i^A(n, \theta) + z_i(n, \theta) \in \{0, 1\}$ for all $(n, \theta) \in [0, 1]^{m+1}$ and, moreover,

$$\int_H z_i(\tilde{n}^h(\omega), \tilde{\theta}^h(\omega)) d\eta(h) = 0 \quad (2.10)$$

for $i = c, 1, \dots, m$ and almost all $\omega \in \Omega$, which by (2.4) is equivalent to the requirement that

$$\int_{[0,1]^{m+1}} z_i(n, \theta) dF(n, \theta) = 0 \quad (2.11)$$

for $i = c, 1, \dots, m$.¹¹ Given A , the net-trade allocation $(z_c(\cdot, \cdot), z_1(\cdot, \cdot), \dots, z_m(\cdot, \cdot))$ is *incentive compatible* if

$$z_c(n, \theta) + \sum_{i=1}^m z_i(n, \theta) \theta_i Q_i^A \geq z_c(n, \theta') + \sum_{i=1}^m z_i(n, \theta') \theta_i Q_i^A \quad (2.12)$$

for all (n, θ) and (n', θ') in $[0, 1]^{m+1}$ for which $\chi_i(n, \theta) + z_i(n', \theta') \in \{0, 1\}$ for all i . The idea is that the holdings $(c^A(n, \theta), \chi_1^A(n, \theta), \dots, \chi_m^A(n, \theta))$ of private-good consumption and public-goods admission tickets of a given agent as well as the realization (n, θ) of $(\tilde{n}, \tilde{\theta})$ are not known by anybody else.¹² Therefore, if the agent claims that the realization of $(\tilde{n}, \tilde{\theta})$ is (n', θ') ,

¹¹In this first approach, I omit the requirement that private-good consumption must not be negative. This device eliminates the interdependence of labour input provision and public-good consumption that arises when people with low labour incomes are unable to pay for the admission tickets to the public goods. This interdependence will be the subject of another study.

¹²One might argue that the mechanism designer knows the consumer's actual holdings, and therefore the incentive constraints may be loosened. Such loosening of incentive constraints would tend to enhance the scope for renegotiations and make the condition of renegotiation proofness even more restrictive. In the large economy considered here, it does not actually make a difference because the characterization of renegotiation proofness in Lemma 2.2 remains valid. In a finite economy, there would be a difference.

he obtains the net trade $(z_c^A(n', \boldsymbol{\theta}'), z_1^A(n', \boldsymbol{\theta}'), \dots, z_m^A(n', \boldsymbol{\theta}'))$ that is available to an agent with parameters $n', \boldsymbol{\theta}'$. Incentive compatibility of the net-trade allocation requires that such a claim must not provide the agent with an improvement over the stipulated net trade $(z_c(n, \boldsymbol{\theta}), z_1(n, \boldsymbol{\theta}), \dots, z_m(n, \boldsymbol{\theta}))$.

An allocation A is said to be *renegotiation proof* if, starting from A , there is no feasible and incentive compatible net-trade allocation which provides a Pareto improvement in the sense that for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, the utility gain from the net trade $(z_c(n, \boldsymbol{\theta}), z_1(n, \boldsymbol{\theta}), \dots, z_m(n, \boldsymbol{\theta}))$ is nonnegative, i.e.

$$z_c(n, \boldsymbol{\theta}) + \sum_{i=1}^m z_i(n, \boldsymbol{\theta}) \theta_i Q_i^A \geq 0, \quad (2.13)$$

and the aggregate utility gain is *strictly* positive, i.e.

$$\int_{[0,1]^{m+1}} [z_c(\tilde{n}^h(\omega), \tilde{\boldsymbol{\theta}}^h(\omega)) + \sum_{i=1}^m z_i(\tilde{n}^h(\omega), \tilde{\boldsymbol{\theta}}^h(\omega)) \tilde{\theta}_i^h(\omega) Q_i^A] dF(n, \boldsymbol{\theta}) > 0, \quad (2.14)$$

with positive probability; by (2.4), the latter inequality is equivalent to the inequality

$$\int_{[0,1]^{m+1}} [z_c(n, \boldsymbol{\theta}) + \sum_{i=1}^m z_i(n, \boldsymbol{\theta}) \theta_i Q_i^A] dF(n, \boldsymbol{\theta}) > 0, \quad (2.15)$$

which actually implies that (2.14) holds with probability one.

The following lemma shows that an allocation is renegotiation proof if and only if there exists a price system which supports the allocation as a competitive equilibrium of the exchange economy in which people trade the private good as well as admission tickets for the different public goods, taking the vector \mathbf{Q}^A of public-good provision levels as given.

Lemma 2.1 *An allocation A is renegotiation proof if and only if there exist prices p_1^A, \dots, p_m^A such that for $i = 1, \dots, m$, and almost all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, one has*

$$\chi_i^A(n, \boldsymbol{\theta}) = 0 \quad \text{if } \theta_i Q_i^A < p_i^A \quad (2.16)$$

and

$$\chi_i^A(n, \boldsymbol{\theta}) = 1 \quad \text{if } \theta_i Q_i^A > p_i^A. \quad (2.17)$$

Renegotiation proofness implies, for each public good i , a simple division between participants with high θ_i and participants with low θ_i . The former get admission to public good i with probability one; the latter do not get admission to public good i at all.

2.4 Using Renegotiation Proofness to Decompose Incentive Compatibility

The prices p_1^A, \dots, p_m^A in Lemma 2.1 can be interpreted as *admission fees*. If the allocation is incentive compatible, as well as renegotiation proof, it must be the case that admission to public good i is granted if and only if the person in question surrenders p_i^A units of private-good consumption. Then agents with $\theta_i Q_i^A > p_i^A$ pay the fee and enjoy the public good for a net benefit equal to $\theta_i Q_i^A - p_i^A$; agents with $\theta_i Q_i^A < p_i^A$ do not pay the fee and are excluded from the public good. Thus one obtains:

Lemma 2.2 *If A is a renegotiation proof and incentive compatible allocation with associated prices p_1^A, \dots, p_m^A , then the expected-payoff function $v^A(\cdot)$ takes the form*

$$v^A(n, \theta_1, \dots, \theta_m) = \bar{v}^A(n) + \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0). \quad (2.18)$$

Moreover, for any $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, the admission indicators $\chi_i^A(n, \boldsymbol{\theta})$ satisfy

$$\chi_i^A(n, \boldsymbol{\theta}) = \bar{\chi}_i^A(\theta_i), \quad (2.19)$$

for $i = 1, \dots, m$, where

$$\bar{\chi}_i^A(\theta_i) = 0 \text{ if } \theta_i Q_i^A < p_i^A \text{ and } \bar{\chi}_i^A(\theta_i) = 1 \text{ if } \theta_i Q_i^A > p_i^A. \quad (2.20)$$

In an allocation that is renegotiation proof *and* incentive compatible, the admission of a person to any one public good i depends *only* on the person's preference parameter θ_i for that particular public good. The person's productivity parameter n and the preference parameters θ_j for public goods $j \neq i$ do not affect the admission to public good i . Admissions decisions for the different public goods are thus separated from each other and from decisions on labour inputs and production.

Given that $\chi_i^A(n, \boldsymbol{\theta})$ satisfies (2.20), the contribution of public-goods enjoyment to the expected payoff $v^A(n, \theta_1, \dots, \theta_m)$ is entirely captured by the surplus $\sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0)$. The difference between $v^A(n, \theta_1, \dots, \theta_m)$ and the surplus $\sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0)$ does *not* depend on the preference parameters $\theta_1, \dots, \theta_m$. This difference can be written as

$$\bar{v}^A(n) = c_0^A(n, \boldsymbol{\theta}) - \gamma(y^A(n, \boldsymbol{\theta}), n), \quad (2.21)$$

where

$$c_0^A(n, \boldsymbol{\theta}) = c^A(n, \boldsymbol{\theta}) + \sum_{i=1}^m \chi_i^A(n, \boldsymbol{\theta}) p_i^A. \quad (2.22)$$

Conceptually, $\bar{v}^A(n)$ is the payoff that an individual with characteristics $n, \boldsymbol{\theta}$ obtains from working the amount $\gamma(y^A(n, \boldsymbol{\theta}), n)$ and consuming the amount $c_0^A(n, \boldsymbol{\theta})$ of the private good without purchasing admission to any of the public goods. Lemma 2.2 indicates that under a renegotiation proof and incentive compatible allocation this payoff is independent of $\boldsymbol{\theta}$.

The following proposition uses this independence property to show that $c_0^A(n, \boldsymbol{\theta})$ and $y^A(n, \boldsymbol{\theta})$ in turn must be independent of $\boldsymbol{\theta}$. There is thus no direct interdependence between the admissions to different public goods i and j or between the admission to some public good i and the provision of labour. The $m + 1$ -dimensional mechanism design problem is transformed into a problem involving $m + 1$ unidimensional incentive constraints.

Proposition 2.3 *An allocation A is renegotiation proof and incentive compatible if and only if there exist prices p_1^A, \dots, p_m^A and functions $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$ from $[0, 1]$ into \mathbb{R}_+ such that the following conditions hold:*

- (a) *the expected payoff function $v^A(\cdot)$ and the admission indicators $\chi_i^A(n, \boldsymbol{\theta})$, $i = 1, \dots, m$, satisfy (2.18) - (2.20);*
- (b) *for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$,*

$$c_0^A(n, \boldsymbol{\theta}) = \hat{c}^A(n) \quad \text{and} \quad y^A(n, \boldsymbol{\theta}) = \hat{y}^A(n), \quad (2.23)$$

$$\hat{c}^A(n) = \bar{v}^A(n) + \gamma(\hat{y}^A(n), n), \quad (2.24)$$

and

$$\bar{v}^A(n) \geq \bar{v}^A(\hat{n}) + \gamma(\hat{y}^A(\hat{n}), \hat{n}) - \gamma(\hat{y}^A(\hat{n}), n) \quad (2.25)$$

for all $\hat{n} \in [0, 1]$.

The proof of this result in the appendix involves two steps. The first step uses Lemmas 2.1 and 2.2 to show that an allocation is renegotiation proof and incentive compatible if and only if it satisfies statement (a) and the incentive constraint

$$\bar{v}^A(n) \geq \bar{v}^A(\hat{n}) + \gamma(y^A(\hat{n}, \boldsymbol{\theta}), \hat{n}) - \gamma(y^A(\hat{n}, \boldsymbol{\theta}), n) \quad (2.26)$$

for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$ and all $\hat{n} \in [0, 1]$. The second step uses (2.26) to prove that $y^A(n, \boldsymbol{\theta})$ must be independent of $\boldsymbol{\theta}$. For any given $\boldsymbol{\theta}$, (2.26) is

exactly the incentive compatibility condition for the unidimensional optimal-income tax problem studied by Mirrlees (1971, 1976). By standard arguments, it follows that $\bar{v}^A(\cdot)$ satisfies the differential equation

$$\frac{d\bar{v}^A}{dn}(n) = -\gamma_n(y^A(n, \boldsymbol{\theta}), n) \quad (2.27)$$

for almost all n and all $\boldsymbol{\theta}$. Because the left-hand side of (2.27) is independent of $\boldsymbol{\theta}$ and because $\gamma_{ny} < 0$, equation (2.27) implies that $y^A(n, \boldsymbol{\theta})$ is independent of $\boldsymbol{\theta}$, which is the point of (2.23).

Given that (2.25) is a unidimensional incentive compatibility condition and given the assumption that $\gamma_{ny} < 0$, the results of Mirrlees (1976) yield:

Remark 2.4 *The incentive compatibility condition (2.25) holds for all n and \hat{n} if and only if the function $\bar{v}^A(\cdot)$ takes the form*

$$\bar{v}^A(n) = \bar{v}^A(0) - \int_0^n \gamma_n(\hat{y}^A(n'), n') dn' \quad (2.28)$$

and the functions $\hat{c}^A(\cdot)$ and $\hat{y}^A(\cdot)$ are nondecreasing.¹³

2.5 Feasibility and the Government Budget Constraint

For any $\mathbf{Q} \in \mathfrak{R}_+^m$, let $K(\mathbf{Q})$ be the aggregate amount of private-good consumption *per capita* that has to be foregone for the vector \mathbf{Q} of public-good provision levels. The cost function $K(\cdot)$ is assumed to be strictly increasing, strictly convex and twice continuously differentiable, with $K(\mathbf{0}) = 0$ and with partial derivatives $K_i(\cdot)$ satisfying $\lim_{k \rightarrow \infty} K_i(\mathbf{Q}^k) = 0$ for any sequence $\{\mathbf{Q}^k\}$ with $\lim_{k \rightarrow \infty} Q_i^k = 0$ and $\lim_{k \rightarrow \infty} K_i(\mathbf{Q}^k) = \infty$ for any sequence $\{\mathbf{Q}^k\}$ with $\lim_{k \rightarrow \infty} Q_i^k = \infty$.

An allocation A is said to be *feasible* if it satisfies

$$\chi_i^A(n, \boldsymbol{\theta}) = 1 \quad (2.29)$$

for $i \in J^{ne}$ and all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, and

$$\frac{1}{\eta(H)} \int_H c^A(\tilde{n}^h(\omega), \tilde{\boldsymbol{\theta}}^h(\omega)) d\eta(h) + K(\mathbf{Q}^A) \leq \frac{1}{\eta(H)} \int_H y^A(\tilde{n}^h(\omega), \tilde{\boldsymbol{\theta}}^h(\omega)) d\eta(h) \quad (2.30)$$

¹³By (2.28) and the monotonicity of $\hat{y}^A(\cdot)$, monotonicity of $\hat{c}^A(\cdot)$ is gratuitous.

for almost all $\omega \in \Omega$, so that the sum of aggregate consumption and public-good provision costs does not exceed aggregate output. By (2.4), (2.30) is equivalent to the requirement that

$$\int_{[0,1]^{m+1}} c^A(n, \boldsymbol{\theta}) dF(n, \boldsymbol{\theta}) + K(\mathbf{Q}^A) \leq \int_{[0,1]^{m+1}} y^A(n, \boldsymbol{\theta}) dF(n, \boldsymbol{\theta}). \quad (2.31)$$

For a renegotiation proof and incentive compatible allocation, Proposition 2.3 implies that (2.29) and (2.31) take the form

$$p_i^A = 0 \quad \text{for } i \in J^{ne} \quad (2.32)$$

and

$$K(\mathbf{Q}^A) \leq \int_0^1 [\hat{y}^A(n) - \hat{c}(n)] dF^n(n) + \sum_{i=1}^m p_i^A (1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A))), \quad (2.33)$$

where, for any i ,

$$\hat{\theta}_i(p_i^A, Q_i^A) := \frac{p_i^A}{Q_i^A} \quad \text{if } Q_i^A > 0, \quad \text{and} \quad \hat{\theta}_i(p_i^A, Q_i^A) := 1 \quad \text{if } Q_i^A = 0. \quad (2.34)$$

In view of (2.20), $\hat{\theta}_i(p_i^A, Q_i^A)$ is defined so that $\chi_i(n, \theta) = 0$ whenever $\theta_i < \hat{\theta}_i(p_i^A, Q_i^A)$, and $\chi_i(n, \theta) = 1$ whenever $\theta_i \in (\hat{\theta}_i(p_i^A, Q_i^A), 1]$.

The second term on the right-hand side of (2.33) corresponds to aggregate revenues from admission fees. Given the fees p_1^A, \dots, p_m^A , for any i , there are $(1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A)))$ participants asking for admission to public good i . Aggregate admission fee revenue from public good i is therefore $p_i^A (1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A)))$, which is positive if $p_i^A \in (0, Q_i^A)$ and zero if $p_i^A = 0$, in particular if the public good is nonexcludable. Aggregate admission fee revenue from *all* public goods is obtained by summing over all $i = 1, \dots, m$.

The first term on the right-hand side of (2.33) corresponds to net aggregate revenue from direct taxation. By the well known taxation principle, this term can be written as a function of the output $\hat{y}^A(n)$ and can be interpreted as an income tax.

Remark 2.5 *If the allocation A is renegotiation proof and incentive compatible, there exists a function $T^A : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that, for any n ,*

$$\hat{y}^A(n) - \hat{c}(n) = T^A(\hat{y}^A(n)), \quad (2.35)$$

and (2.33) takes the form

$$K(\mathbf{Q}^A) \leq \int_0^1 T^A(\hat{y}^A(n)) dF^n(n) + \sum_{i=1}^m p_i^A (1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A))). \quad (2.36)$$

For almost every $n \in [0, 1]$, the derivative of $T^A(\cdot)$ at the point $\hat{y}^A(n)$ is well defined and satisfies

$$\frac{dT^A}{dy}(\hat{y}^A(n)) = 1 - \gamma_y(\hat{y}^A(n), n). \quad (2.37)$$

The feasibility constraint thus takes the form of a government budget constraint requiring that the cost $K(\mathbf{Q}^A)$ of public-goods provision be covered by the sum of total revenues from income taxes and from admission fees for public goods. If net aggregate revenue from direct taxation is negative, e.g., because, for low values of n , the mechanism designer is providing a subsidy rather than imposing a tax, then the net aggregate subsidy $-\int_0^1 T^A(\hat{y}^A(n)) dF^n(n)$ must not exceed the surplus of admission fee revenues over the costs of public-goods provision.

3 Welfare Maximizing Allocations

3.1 First-Best Allocations

Turning to the analysis of welfare-maximizing allocations, I begin with *first-best* allocations, i.e. allocations which maximize (2.6) over the set of feasible allocations, without regard for renegotiation proofness and incentive compatibility. By standard arguments, one obtains:

Proposition 3.1 *Let $y^* : [0, 1] \rightarrow \mathfrak{R}_+$ and $\mathbf{Q}^* \in \mathfrak{R}_+^m$ be such that*

$$\gamma_y(y^*(n), n) = 1 \quad (3.1)$$

for all n and

$$\int_0^1 \theta_i dF^i(\theta_i) = K_i(\mathbf{Q}^*) \quad (3.2)$$

for $i = 1, \dots, m$. An allocation A is first-best if and only if it satisfies the feasibility condition (2.31) with equality as well as $\mathbf{Q}^A = \mathbf{Q}^$, and*

$$y^A(n, \boldsymbol{\theta}) = y^*(n), \quad (3.3)$$

$$\chi_i^A(n, \boldsymbol{\theta}) = 1, \quad (3.4)$$

and, for some $\lambda > 0$,

$$W' \left(c^A(n, \boldsymbol{\theta}) + \sum_{i=1}^m \theta_i Q_i^* - \gamma(y^*(n), n) \right) = \lambda, \quad (3.5)$$

for almost all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$.

Under the given assumptions about the cost functions γ and K , first-best levels of public-good provision and of output provision by individuals are unique and are strictly positive. The ability to exclude people from the enjoyment of a public good is never utilized. First-best levels of public-good provision are chosen so that for each i , the marginal cost $K_i(\mathbf{Q}^A)$ of increasing Q_i is equal to the aggregate marginal benefit that consumers draw from the increase; this is the well-known Lindahl-Samuelson condition. First-best output levels depend only on the individuals' productivity parameters; output provision is driven to the point where marginal cost is equal to one.

Social marginal benefits of consumption are equalized across participants. If the welfare function W is strictly concave, this implies that, for some constant c_0^* , one has

$$c^A(n, \boldsymbol{\theta}) = c_0^* + \gamma(y^*(n), n) - \sum_{i=1}^m \theta_i Q_i^* \quad (3.6)$$

for almost all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, so the dependence of private-good consumption on n and $\boldsymbol{\theta}$ is used to compensate for output provision costs and public-goods enjoyment, with the consequence that $v^A(n, \boldsymbol{\theta}) = c_0^*$, regardless of n and $\boldsymbol{\theta}$.

In combination with (3.4), equation (3.6) is incompatible with incentive compatibility: By (3.4) and Lemma 2.1, a first-best allocation is renegotiation proof. By Lemma 2.2 therefore, incentive compatibility is violated if $c^A(n, \boldsymbol{\theta})$ depends on $\boldsymbol{\theta}$ as shown in (3.6). Proposition 3.1 thus has the following corollary.

Corollary 3.2 *If the welfare function W is strictly concave, a first-best allocation is not incentive compatible.*

In contrast, if the welfare function W is affine, condition (3.5) has no bite. In this case, Proposition 3.1 yields:

Corollary 3.3 *If the welfare function W is affine, a first-best allocation is incentive compatible if and only if it satisfies*

$$c^A(n, \theta) = y^*(n) - K(\mathbf{Q}^*). \quad (3.7)$$

for all (n, θ) , $\mathbf{Q}^A = \mathbf{Q}^*$, $y^A(n, \theta) = y^*(n)$ for all n and $\chi_i^A(n, \theta) = 1$.

In the absence of inequality aversion, a first-best allocation is implemented by levying the lump-sum tax $K(\mathbf{Q}^*)$ on everybody; in this case, optimal admission fees and marginal income tax rates are equal to zero.

3.2 Second-Best Allocations

This section considers *second-best allocations*, defined as those allocations which maximize the welfare functional (2.6) over the set of all feasible, incentive compatible and renegotiation proof allocations. By Proposition 2.3 and Remark 2.4, the problem of choosing a second-best allocation is equivalent to the problem of choosing public-goods provision levels Q_i^A for $i = 1, \dots, m$, admission prices p_i^A for $i \in J^e$, an expected-payoff function $\bar{v}^A(\cdot)$ and nondecreasing functions $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$ so as to maximize

$$\int_{[0,1]^{m+1}} W \left(\bar{v}^A(n) + \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0) \right) dF(n, \theta) \quad (3.8)$$

subject to the feasibility constraint

$$K(Q_1^A, \dots, Q_m^A) \leq \int_0^1 [\hat{y}^A(n) - \hat{c}^A(n)] dF^n(n) + \sum_{i=1}^m p_i^A (1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A))) \quad (3.9)$$

and the conditions that $p_i^A = 0$ for $i \in J^{ne}$, and, for any $n \in [0, 1]$,

$$\hat{c}^A(n) = \bar{v}^A(n) + \gamma(\hat{y}^A(n), n) \quad (3.10)$$

and

$$\bar{v}^A(n) = \bar{v}^A(0) - \int_0^n \gamma_n(\hat{y}^A(n'), n') dn'. \quad (3.11)$$

Solutions to the latter problem are characterized by standard techniques. The following proposition indicates the first-order necessary conditions for a maximum.

Proposition 3.4 *Let A be a second-best allocation, let p_1^A, \dots, p_m^A be the associated admissions prices, let $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$ be the associated consumption, output provision and payoff functions, and let*

$$\lambda := \int_{[0,1]^{m+1}} W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}). \quad (3.12)$$

Then, for any nonexcludable public good $i \in J^{ne}$,

$$\int_{[0,1]^{m+1}} W'(v^A(n, \boldsymbol{\theta})) \theta_i dF(n, \boldsymbol{\theta}) = \lambda K_i(\mathbf{Q}^A). \quad (3.13)$$

For any excludable public good $i \in J^e$,

$$\int_{[0,1]^{m+1}} W'(v^A(n, \boldsymbol{\theta})) \max(\theta_i - \hat{\theta}_i^A, 0) dF(n, \boldsymbol{\theta}) + \lambda \hat{\theta}_i^A (1 - F^i(\hat{\theta}_i^A)) = \lambda K_i(\mathbf{Q}^A) \quad (3.14)$$

and

$$\lambda (1 - F^i(\hat{\theta}_i^A) - \hat{\theta}_i^A f^i(\hat{\theta}_i^A)) - \int_{\hat{\theta}_i^A}^1 \int_{[0,1]^m} W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) = 0, \quad (3.15)$$

where $\hat{\theta}_i^A := \hat{\theta}_i(p_i^A, Q_i^A)$, as defined in (2.34).

Moreover, for any $n \in [0, 1]$,

$$\int_n^1 [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n')) f^n(n') + \gamma_{ny}(\hat{y}^A(n'), n') \psi^A(n')] dn' \geq 0, \quad (3.16)$$

where $\psi^A(\cdot)$ is an absolutely continuous function satisfying

$$\psi^A(n) = \int_n^1 \int_{[0,1]^m} (W'(v^A(n, \boldsymbol{\theta})) - \lambda) dF(n', \boldsymbol{\theta}) \quad (3.17)$$

for all n . If the inequality in (3.16) is strict, then $\hat{y}^A(\cdot)$ is constant on some open neighbourhood of n . If $\hat{y}^A(\cdot)$ is strictly increasing at n , (3.16) holds as an equation; therefore,

$$\lambda(1 - \gamma_y(\hat{y}^A(n), n)) f^n(n) = \gamma_{ny}(\hat{y}^A(n), n) \psi^A(n) \quad (3.18)$$

if, on some open neighbourhood of n , the monotonicity constraint on $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$ is not binding.

Because renegotiation proofness has reduced the general $m+1$ -dimensional mechanism design problem to a problem with $m+1$ unidimensional incentive constraints, the first-order conditions for a second-best allocation have the same structure as the corresponding conditions in the unidimensional utilitarian public-good provision problem and the unidimensional utilitarian income tax problem.¹⁴ The multidimensional nature of the problem does however appear in the term $W'(v^A(n, \theta))$ in (3.12) - (3.15) and (3.17). This term indicates the marginal social welfare attached to an additional unit of private-good consumption for a person with productivity and taste parameters n and $\theta_1, \dots, \theta_m$. It depends in a nontrivial way on all these parameters. (3.13) - (3.15) therefore require taking expectations with respect to n and θ_{-i} , i.e. those parameters which are not directly relevant for public good i ; similarly, (3.17) involves taking expectations with respect to the vector θ of those parameters which are not directly relevant for labour-leisure choices. If the different parameters are mutually independent, i.e. if F takes the form of a product $F^n \times F^1 \times \dots \times F^m$, this integration has no effect on the underlying tradeoffs, and the first-order conditions have *exactly* the same structure as in the corresponding unidimensional problems. However, if the different parameters are *not* independent, the correlations can affect the underlying tradeoffs.

To see why, consider the standard argument for the positivity of the optimal marginal income tax rate at $\hat{y}^A(n)$ for $n \in (0, 1)$. By Remark 2.5, the optimal marginal income tax rate at $\hat{y}^A(n)$ is strictly positive if $\gamma_y(\hat{y}^A(n), n) < 1$. Under the assumption that W is strictly concave, Mirrlees (1971, 1976) and Seade (1977, 1982) derive this result from (3.18) and the observation that the costate variable $\psi^A(n)$ is negative for $n \in (0, 1)$. Negativity of $\psi^A(n)$ for $n \in (0, 1)$ is obtained from the unidimensional analogues of (3.12) and (3.17) in combination with the observation that $W'(\bar{v}^A(n))$ is strictly decreasing in n .¹⁵ In the multidimensional setting considered here, the negativity of $\psi^A(n)$ for $n \in (0, 1)$ does not follow so easily: Rewrite

¹⁴For the unidimensional utilitarian public-good provision problem, see Hellwig (2003); for the utilitarian income tax problem, Hellwig (2004 b). Conditions (3.18) and (3.17) are essentially the conditions of Mirrlees (1971, 1976) and Seade (1977, 1982) for the case where the monotonicity constraint is not binding. (3.16) generalizes the condition of Ebert (1992) and Brunner (1993), to allow for the possibility that $\hat{y}^A(\cdot)$ may not be continuous, let alone piecewise continuously differentiable.

¹⁵The latter observation follows from strict concavity of $W(\cdot)$ in combination with the fact that incentive compatibility implies strict monotonicity of $\bar{v}^A(\cdot)$.

(3.17) in the form

$$\psi^A(n) = \int_n^1 \left[\int_{[0,1]^m} W'(v^A(n', \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n') - \lambda \right] dF^n(n'), \quad (3.19)$$

where $F(.|n')$ is the conditional distribution of $\tilde{\boldsymbol{\theta}}$ given $\tilde{n} = n'$. The usual argument for the negativity of $\psi^A(n)$ for $n \in (0, 1)$ is valid if the function

$$n' \rightarrow \int_{[0,1]^m} W'(v^A(n', \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n') \quad (3.20)$$

is decreasing. However, this monotonicity property is *not* ensured merely by the monotonicity of the integrand. If the productivity parameter were negatively correlated with the taste parameters, the dependence of the conditional distribution $F(.|n')$ on n' could outweigh the monotonicity of the integrand, at least locally.

Monotonicity of (3.20) and negativity of $\psi^A(n)$ for $n \in (0, 1)$ are ensured if productivity and taste parameters are independent or if they are positively correlated. The appropriate notion of positive correlation here is provided by the concept of *affiliation*. As defined in Milgrom and Weber (1982), the random variables $\tilde{n}, \tilde{\theta}_1, \dots, \tilde{\theta}_m$ are *affiliated* if and only if their joint density f satisfies the inequality

$$f((n, \boldsymbol{\theta}) \vee (n', \boldsymbol{\theta}')) \cdot f((n, \boldsymbol{\theta}) \wedge (n', \boldsymbol{\theta}')) \geq f((n, \boldsymbol{\theta})) \cdot f((n', \boldsymbol{\theta}'))$$

for all $(n, \boldsymbol{\theta})$ and $(n', \boldsymbol{\theta}')$ in $[0, 1]^{m+1}$, where $(n, \boldsymbol{\theta}) \vee (n', \boldsymbol{\theta}')$ and $(n, \boldsymbol{\theta}) \wedge (n', \boldsymbol{\theta}')$ refer to the vectors of component-wise maxima and component-wise minima of $(n, \boldsymbol{\theta})$ and $(n', \boldsymbol{\theta}')$.

Proposition 3.5 *Let A be a second-best allocation, with associated consumption, output provision and payoff functions $\hat{c}^A(.), \hat{y}^A(.), \bar{v}^A(.)$. If the productivity and taste parameters are affiliated and if the welfare function W is strictly concave, then $\gamma_y(\hat{y}^A(n), n) < 1$ and $\hat{y}^A(n) < y^*(n)$ for all $n \in (0, 1)$. Moreover, $\lim_{n \uparrow 1} \gamma_y(\hat{y}^A(n), n) = 1$ and $\lim_{n \uparrow 1} \hat{y}^A(n) = y^*(1)$. If the monotonicity constraint on $\hat{y}^A(.)$ is nonbinding in a neighbourhood of zero, $\lim_{n \downarrow 0} \gamma_y(\hat{y}^A(n), n) = 1$ and $\lim_{n \downarrow 0} \hat{y}^A(n) = y^*(0)$.*

Corollary 3.6 *Under the assumptions of Proposition 3.5, the optimal marginal income tax is positive at $\hat{y}^A(n)$ for $n \in (0, 1)$, zero at $\lim_{n \uparrow 1} \hat{y}^A(n)$ and zero at $\lim_{n \downarrow 0} \hat{y}^A(n)$ unless the monotonicity constraint on $\hat{y}^A(.)$ is binding in a neighbourhood of zero.*

The first-order conditions (3.13) and (3.14) for public-goods provision levels correspond to a modified Lindahl-Samuelson condition. For a nonexcludable public good, Q_i^A is determined in such a way that the marginal provision cost is equal to a marginal-welfare-weighted aggregate of the marginal benefits that consumers draw from an increase in Q_i^A . For an excludable public good, the marginal provision cost is equated to the sum of the marginal-welfare-weighted aggregate of the marginal benefits that are obtained by users and the aggregate marginal revenues that are obtained by the mechanism designer if the admission fee p_i^A is raised in proportion to Q_i^A so that the critical $\tilde{\theta}_i(p_i^A, Q_i^A)$ is unchanged.

Under the affiliation assumption, the total marginal benefits of an increase in Q_i^A are in both cases less than $\int_0^1 \theta_i dF^i(\theta_i)$, the total marginal benefit of an increase in Q_i^A in a first-best setting. If the cost function K is additively separable, it follows that second-best levels of public-goods provision are lower than first-best levels.

For a nonexcludable public good, the difference is due to the negative correlation between the taste parameter $\tilde{\theta}_i$ and the conditional expectation, given θ_i , of the marginal welfare weight W' ; this negative correlation implies that high values of the taste parameter receive relatively less weight. If $\tilde{n}, \tilde{\theta}_1, \dots, \tilde{\theta}_m$ are independent, the negative correlation reflects the mechanism designer's aversion against inequality of payoffs induced by differences in θ_i ; this effect has been discussed in Hellwig (2003). If $\tilde{n}, \tilde{\theta}_1, \dots, \tilde{\theta}_m$ are positively correlated, the negative correlation between $\tilde{\theta}_i$ and W' also reflects the mechanism designer's aversion against inequality of payoffs induced by differences in earning abilities, as in Boadway and Keen (1993).¹⁶

For an excludable public good, the difference between the conditions for second-best and first-best public-goods provision is also due to the fact that there are fewer users of the public good than in the first-best allocation. Moreover, the mechanism designer is unable to fully appropriate the benefits from additional public-good provision so aggregate marginal revenues accruing to him are less than aggregate marginal benefits accruing to users. From these considerations, one obtains:

¹⁶In the present model, with additively separable payoffs, correlations between the parameters $\tilde{n}, \tilde{\theta}_1, \dots, \tilde{\theta}_m$ take the place of nonseparabilities in Boadway and Keen (1993). Independence of $\tilde{n}, \tilde{\theta}_1, \dots, \tilde{\theta}_m$ corresponds to the separable specifications in Boadway and Keen (1993), as well as Christiansen (1981). In contrast to their results, independence here does not yield first-best provision levels because of the combination of nondegeneracy of $\tilde{\theta}_i^h$ and inequality aversion. Independence would yield first-best provision levels if the mechanism designer was able to observe the realizations of $\tilde{\theta}_i^h$.

Proposition 3.7 *Let A be a second-best allocation. If the productivity and taste parameters are affiliated and if the welfare function W is strictly concave, the vector \mathbf{Q}^A of public-goods provision levels satisfies $\mathbf{Q}^A \gg 0$ and*

$$\int_0^1 \theta_i dF^i(\theta_i) > K_i(\mathbf{Q}^A)$$

for $i = 1, \dots, m$. If $K(\cdot)$ is additively separable, second-best public-goods provision levels are strictly less than their first-best counterparts.

As for the admission fees p_i^A for $i \in J^e$, an increase in p_i^A reduces the private-good consumption of people with $\theta_i > \hat{\theta}_i^A$ who are paying the higher fee while at the same time raising admission fee revenues and permitting an equal increase in everybody's private-good consumption. Condition (3.15) balances these two effects at the margin. Upon rewriting this condition in the form

$$\begin{aligned} \hat{\theta}_i^A f^i(\hat{\theta}_i^A) &= \lambda (1 - F^i(\hat{\theta}_i^A)) - \int_{\hat{\theta}_i^A}^1 \int_{[0,1]^m} W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) \\ &= \int_{\hat{\theta}_i^A}^1 \left[\lambda - \int_{[0,1]^m} W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}_{-i} | \theta_i) \right] dF^i(\theta_i) \end{aligned} \quad (3.21)$$

one sees that it involves the same kind of equity-efficiency tradeoff as conditions (3.16) - (3.18): In (3.21), the efficiency loss of a marginal price increase inducing people with taste parameter value $\theta_i = \hat{\theta}_i^A = \frac{p_i^A}{Q_i^A}$ to cease demanding admission to the public good is equated to the welfare gain from the redistribution that is induced by this price increase. (3.22) shows that the redistribution indeed involves a welfare gain rather than a welfare loss if the additional welfare attached to a marginal increase in private good consumption is lower, on average, for people with $\theta_i > \hat{\theta}_i^A$ than for the population as a whole. Here again, one has to be careful about correlations and their effects on the conditional expectation $\int_{[0,1]^m} W' dF(n, \boldsymbol{\theta}_{-i} | \theta_i)$ in (3.22). However, if the productivity and taste parameters are affiliated, the right-hand side of (3.22) is nonnegative, indicating a welfare gain rather than a welfare loss from the marginal redistribution effect.

However, even if the affiliation assumption on productivity and taste parameters is satisfied, the equity-efficiency tradeoff in (3.22) does *not* necessarily call for distortionary admission fees. In this respect, the formal similarity of the equity-efficiency tradeoffs in (3.22) and in (3.16) - (3.18)

hides an underlying difference.¹⁷ In Corollary 3.6, the positivity of the optimal marginal income tax for $n \in (0, 1)$ is obtained from the familiar argument that, starting from an efficient allocation, the introduction of a small wedge into leisure-consumption choices has first-order effects on redistribution gains and second-order effects on efficiency losses. For admission fees, this argument is not available, because, when starting from an efficient allocation, i.e. an allocation involving zero admission fees, the effects of introducing a small admission fee for public good i on redistribution gains are of the same order of magnitude as the effects on efficiency losses from having fewer people enjoy the public good. The reason is that, for $p_i^A = \hat{\theta}_i^A = 0$, the redistribution effect on the right-hand side of (3.22) is zero because, up to a set of measure zero, the set of people with $\theta_i > \hat{\theta}_i^A$ coincides with the population as a whole. The first-order effects of small admission fees are thus equal to zero, for redistribution gains as well as efficiency losses. The desirability of positive admission fees cannot be assessed simply from the first-order condition (3.15) or (3.22); it requires an investigation of second-order conditions or, more generally, the global properties of the objective function (3.8). As in Hellwig (2003), this investigation leads to the conclusion that *positive admission fees are undesirable* if inequality aversion, i.e. the curvature $\rho_W(v) = -\frac{W''(v)}{W'(v)}$ of the welfare function $W(\cdot)$, is uniformly small; in contrast, *positive admission fees are desirable* if inequality aversion is uniformly large. These findings are summarized in the following result.

Proposition 3.8 (a) *Let $\{W_k\}$ be any sequence of increasing, concave, and twice continuously differentiable functions on \mathfrak{R} such that $\lim_{k \rightarrow \infty} \rho_{W_k}(v) = 0$, uniformly in v , and let $\{A^k\}$ be an associated sequence of second-best allocations. Then $p_i^{A^k} = 0$ for any sufficiently large k and $\lim_{k \rightarrow \infty} Q_i^{A^k} = Q_i^*$.*

(b) *Let $\{W_k\}$ be any sequence of increasing, concave, and twice continuously differentiable functions on \mathfrak{R} such that $\lim_{k \rightarrow \infty} \rho_{W_k}(v) = \infty$, uniformly in v , and let $\{A^k\}$ be an associated sequence of second-best allocations. If A^∞ is any limit point of the sequence $\{A^k\}$, then the pair $(\mathbf{Q}^{A^\infty}, \hat{\theta}^{A^\infty}) = (Q_1^{A^\infty}, \dots, Q_m^{A^\infty}, \hat{\theta}_1(p_1^{A^\infty}, Q_1^{A^\infty}), \dots, \hat{\theta}_m(p_m^{A^\infty}, Q_m^{A^\infty}))$ is a solu-*

¹⁷As far as I can tell, the difference is due to the admission indicator χ_i^A entering the contribution $\chi_i^A \theta_i Q_i^A$ of public good i to the agent's payoff in a linear fashion. Renegotiation proofness and the induced zero-one structure of the admission rule $\chi_i^A(\cdot, \cdot)$ do not seem play a role: Hellwig (2003) contains an example where $\chi_i^A(\cdot) \equiv 1$ even though renegotiation proofness is not imposed and nondegenerate randomization is admitted.

tion to the monopoly problem

$$\max_{(\mathbf{Q}^M, \hat{\boldsymbol{\theta}}^M)} \sum_{i=1}^m [\hat{\theta}_i^M Q_i^M (1 - F_i(\hat{\theta}_i^M)) - K(\mathbf{Q}^M)]. \quad (3.23)$$

In particular, for any $i \in J^e$, $p_i^{A^k} > 0$ for any sufficiently large k .

The implications of Proposition 3.8 for the respective roles of admission fees and direct taxes in public-goods finance are formulated as:

Corollary 3.9 *Let A be a second-best allocation, and let $T^A(\cdot)$ be the associated income tax schedule.*

(a) *If $\rho_W(v)$ is small, uniformly in v , then*

$$K(\mathbf{Q}^A) = \int_0^1 T^A(\hat{y}^A(n)) dF^n(n), \quad (3.24)$$

i.e. public-goods provision is financed entirely from direct taxes.

(b) *If $\rho_W(v)$ is large, uniformly in v , then*

$$\sum_{i=1}^m p_i^A (1 - F_i(\hat{\theta}_i(p_i^A, Q_i^A))) > K(\mathbf{Q}^A), \quad (3.25)$$

i.e. public-goods provision is a money-making venture, with revenues exceeding costs, providing additional resources for redistribution.

If inequality aversion is small, public goods should be entirely financed from direct taxes, and the conclusion of Atkinson and Stiglitz (1976) is confirmed. If inequality aversion is large, admission fees on excludable public goods should be more than enough to cover costs. Indeed, in this case, second-best allocations are close to Rawlsian allocations, which maximize the payoff $v^A(0, \boldsymbol{\theta}) = \hat{c}^A(0) - \gamma(\hat{y}^A(0), 0)$ of the worst-off person in the economy. In a Rawlsian allocation, public-goods provision is managed as a profit-maximizing monopoly so as to maximize (for given $\hat{y}^A(0)$) the amount of private-good consumption $\hat{c}^A(0)$ that can be made available to people with productivity and taste parameters all equal to zero.

Proposition 3.8 and Corollary 3.9 hold regardless of correlations between the different parameters. In particular, they hold if all productivity and taste parameters are stochastically independent, so utilitarian welfare maximization may call for nonzero admission fees for excludable public goods even

when there is no relation between public-goods preferences and productivity levels. The difference between this finding and the results of Atkinson and Stiglitz (1976) is due to redistributive concerns generated by the heterogeneity in public-goods preferences themselves.

Correlations between the different parameters matter for the structure of admission fees and income taxes. because, as discussed above, they do affect the first-order conditions (3.15) and (3.16) - (3.18). I conjecture that, if the correlations between the different productivity and taste parameters are everywhere strictly positive in the sense of the affiliation assumption, then (at least some) admission fees should be higher than under independence.

3.3 Participation Constraints and the Generalized Ramsey-Boiteux Problem

In addition to incentive compatibility, renegotiation proofness and feasibility, I now impose the requirement that the allocation be individually rational, so everybody is willing to participate voluntarily, and the mechanism designer does not have to rely on the government's power of coercion. This additional requirement will add another perspective to the relation between income taxation and public-goods provision and pricing. In Hellwig (2004 a), I had shown that, in a large economy with exogenous production, the imposition of a participation constraint turns the condition for feasibility into a government budget constraint requiring that the costs of public-goods provision be covered by the payments that people are willing to make in order to avoid exclusion from those public goods where exclusion is feasible. With a requirement of renegotiation proofness as well as incentive compatibility, the resulting mechanism design problem was shown to be equivalent to the Ramsey-Boiteux problem of choosing a vector of admission fees subject to the requirement that admission fee revenues suffice to cover the costs of public-goods provision. Here I study how these implications of the individual-rationality requirement are affected by the availability of nonlinear income taxation as an additional source of finance.

The precise meaning of an individual-rationality requirement depends on what alternatives and what payoffs are considered to be available outside the proposed allocation. Here, I assume that people's payoffs outside the proposed allocation are equal to zero. If one person vetoes the allocation, no alternative scheme is put into place, no production activity occurs, and no public goods are provided.

This specification is somewhat stark. Specifications with rosier alternatives to the proposed allocations may seem more plausible, e.g. specifications

which allow people to produce the private good on their own. However, the rosier the alternatives to the proposed allocation are, the more restrictive the individual-rationality constraint is going to be. The specification that is chosen here should be interpreted as a point of reference, indicating the implications of imposing individual rationality in its least restrictive form.

Formally, an allocation A is said to be *individually rational* if the induced payoffs satisfy

$$v^A(n, \theta) \geq 0 \quad (3.26)$$

for all $(n, \theta) \in [0, 1]^{m+1}$. If A is incentive compatible, $v^A(., .)$ is nondecreasing, so (3.26) takes the form $v^A(0, \mathbf{0}) \geq 0$. If A is also renegotiation proof, this latter inequality in turn reduces to

$$\bar{v}^A(0) = \hat{c}^A(0) - \gamma(\hat{y}^A(0), 0) \geq 0, \quad (3.27)$$

where $\bar{v}^A(0)$, $\hat{c}^A(0)$, and $\hat{y}^A(0)$ are given by Lemma 2.2 and Proposition 2.3. Equivalently, the tax schedule $T^A(., .)$ that is given by Remark 2.5 satisfies

$$T^A(\hat{y}^A(0)) \leq \hat{y}^A(0) - \gamma(\hat{y}^A(0), 0), \quad (3.28)$$

i.e. the direct tax on people with productivity parameter zero does not exceed the surplus generated by these people's production. If this surplus is zero, this amounts to a requirement that $K(\mathbf{Q}^A)$ be financed by income taxes and admission fees, without any recourse to lump-sum payments from participants at all.

An allocation is said to be *third-best* if it maximizes the welfare functional (2.6) over the set of feasible, incentive compatible, renegotiation proof and individually rational allocations. By the same argument as before, the problem of choosing a third-best allocation is equivalent to the problem of choosing public-goods provision levels Q_i^A for $i = 1, \dots, m$, admission prices p_i^A for $i \in J^e$, an expected-payoff function $\bar{v}^A(., .)$ and nondecreasing functions $\hat{c}^A(., .)$, $\hat{y}^A(., .)$ so as to maximize (3.8) subject to (3.9) - (3.11) and the participation constraint (3.27). The following result provides an immediate analogue of Proposition 3.4.

Proposition 3.10 *Let A be a third-best allocation, let p_1^A, \dots, p_m^A be the associated admissions prices and let $\hat{c}^A(., .)$, $\hat{y}^A(., .)$, $\bar{v}^A(., .)$ be the associated consumption, output provision and payoff functions. Then there exists a scalar λ with*

$$\lambda \geq \int_{[0,1]^{m+1}} W'(v^A(n, \theta)) \, dF(n, \theta) \quad (3.29)$$

for which A , p_1^A, \dots, p_m^A , and $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$ satisfy the conclusions of Proposition 3.4, i.e. conditions (3.13) - (3.19). Moreover, λ satisfies (3.29) with equality if $\bar{v}^A(0) > 0$.

Thus the imposition of the participation constraint (3.26) leaves the basic structure of the first-order conditions for an optimal allocation unchanged. The only change concerns the Lagrange multiplier for the feasibility constraint: When the participation constraint is binding, the feasibility constraint is harder to meet and has a higher shadow price. The mathematical structure of the first-order conditions is unaffected, but the enhanced weight of the feasibility constraint does introduce a qualitatively new consideration into the determination of income taxes and admission fees. The difference is highlighted by the following result.

Proposition 3.11 *Let A be a third-best allocation, let p_1^A, \dots, p_m^A be the associated admissions prices and let $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$ be the associated consumption, output provision and payoff functions, and suppose that the inequality in (3.29) is strict. Then $p_i^A > 0$ for all $i \in J^e$. Moreover, there exists $\hat{n} > 0$ such that $\gamma_y(\hat{y}^A(n), n) < 1$ and $\hat{y}^A(n) < y^*(n)$, i.e. the optimal marginal income tax rate is positive at $\hat{y}^A(n)$ for all $n \in [0, \hat{n}]$.¹⁸*

Proposition 3.11 should be compared to Propositions 3.5 and 3.8. For a second-best allocation, Proposition 3.5 indicates that, for people with $n = 0$, in the absence of bunching, the optimal marginal income tax is equal to zero. As discussed by Seade (1977), the reason is that, if we think of the marginal income tax as a device providing for an additional levy on all people with higher incomes to be used for redistribution to all people with lower incomes, then, because there is nobody with an income below $\hat{y}^A(0)$, there is no point in using this device at $\hat{y}^A(0)$, i.e. the marginal income tax at zero should be zero. Here the additional levy on all people with higher incomes that is induced by a positive marginal income tax at $\hat{y}^A(0)$ makes it possible to raise $\bar{v}^A(0)$, i.e. to alleviate the participation constraint, or, equivalently, to alleviate the difficulty of meeting the feasibility constraint when the participation constraint restricts the use of lump sum taxation.

Similarly, part (a) of Proposition 3.8 indicates that, in a second-best allocation, the use of admission fees as a redistribution device is undesirable if inequality aversion is small. In this case, efficiency losses from such fees

¹⁸If \tilde{n} and $\tilde{\theta}_1, \dots, \tilde{\theta}_m$ are affiliated, $\hat{n} = 1$.

are always larger than redistribution gains. The comparison is unaffected by the fact that, for small values of the fees, efficiency losses are of the second order of smalls; for small values of the fees, redistribution gains are of the second order of smalls as well. In contrast, in a third-best setting, for small values of the fees, there is a first-order gain from using admission fee revenues to alleviate participation and feasibility constraints.

The participation constraint thus changes the role of the feasibility constraint or, equivalently, the government budget constraint. Whereas, in a second-best setting, the implications of any change, for instance, a reduction in an admission fee, on government revenue can be neutralized by a lump sum change in private-good consumption, inducing an equal change in $\bar{v}^A(n)$ for all n ; in a third-best setting, this device is not available. A lump sum reduction in private-good consumption may run afoul of the participation constraint, in which case, the implications of a reduction in an admission fee on government revenue must be neutralized by another measure. One is then not just trading off the efficiency effects and distribution effects of one distortionary measure, neutralized by a lump sum change in private-good consumption; as in the Ramsey-Boiteux approach to indirect taxation and public-sector pricing, one is instead trading off the efficiency and distribution effects of *different* distortionary measures at the margin. This effect is reflected in the Lagrange multiplier of the feasibility constraint exceeding $\int W'(v^A(n, \theta)) dF(n, \theta)$ when the participation constraint is binding.

The difference between third-best and second-best allocations is most striking when the welfare function $W(\cdot)$ is affine and there is no inequality aversion. For this case, Corollary 3.3 indicates that first-best and second-best allocations coincide and that the participation constraint (3.26) is satisfied if and only if

$$K(\mathbf{Q}^*) \leq y^*(0) - \gamma(y^*(0), 0), \quad (3.30)$$

i.e. if and only if the surplus generated by a person with productivity parameter $n = 0$ exceeds the costs of first-best public-goods provision. In this case, a lump sum tax $K(\mathbf{Q}^*)$ can be imposed without violating the participation constraint; so neither distortionary income taxation nor admission fees are needed for public-goods finance. If (3.30) does not hold, the imposition of a lump sum tax equal to $K(\mathbf{Q}^*)$ is incompatible with individual rationality. In this case, one obtains:

Proposition 3.12 *Assume that $W(\cdot)$ is affine, and let A be a third-best allocation. Let p_1^A, \dots, p_m^A be the associated admissions prices and let $\hat{c}^A(\cdot)$,*

$\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$ be the associated consumption, output provision and payoff functions. If $K(\mathbf{Q}^*) > y^*(0) - \gamma(y^*(0), 0)$, there exists $\lambda > 1$ such that, for $i = 1, \dots, m$,

$$\int_{\hat{\theta}_i^A}^1 (\theta_i - \hat{\theta}_i^A) dF_i(\theta_i) + \lambda \hat{\theta}_i^A (1 - F_i(\hat{\theta}_i^A)) = \lambda K_i(\mathbf{Q}^A) \quad (3.31)$$

and, for any $i \notin J^{ne}$,

$$p_i^A f_i(\hat{\theta}_i^A) \frac{1}{Q_i^A} = \frac{\lambda - 1}{\lambda} (1 - F_i(\hat{\theta}_i^A)), \quad (3.32)$$

where again $\hat{\theta}_i^A = \hat{\theta}_i(p_i^A, Q_i^A)$. Moreover,

$$\int_n^1 \left[-(1 - \gamma_y(\hat{y}^A(n'), n')) f^n(n') + \gamma_{ny}(\hat{y}^A(n'), n') \frac{(1 - \lambda)}{\lambda} (1 - F^n(n')) \right] dn' \geq 0 \quad (3.33)$$

for all $n \in [0, 1]$. If the inequality in (3.33) is strict, then $\hat{y}^A(\cdot)$ is constant on some open neighbourhood of n . If $\hat{y}^A(\cdot)$ is strictly increasing at n , the inequality in (3.33) holds as an equation; in particular,

$$(1 - \gamma_y(\hat{y}^A(n), n)) f^n(n) = \frac{\lambda - 1}{\lambda} (1 - F^n(n)) (-\gamma_{ny}(\hat{y}^A(n), n)) \quad (3.34)$$

if, on some open neighbourhood of n , the monotonicity constraint on $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$ is not binding.

Corollary 3.13 *Under the assumptions of Proposition 3.12, $\gamma_y(\hat{y}^A(n), n) < 1$ and $\hat{y}^A(n) < y^*(n)$ for all $n \in [0, 1]$. Moreover, $\lim_{n \rightarrow 1} \gamma_y(\hat{y}^A(n), n) = 1$ and $\lim_{n \rightarrow 1} \hat{y}^A(n) = y^*(1)$. Further, $p_i^A > 0$ for all $i \notin J^{ne}$ and $K_i(\mathbf{Q}^A) < \int_0^1 \theta_i dF^i(\theta_i)$ for all i ; if $K(\cdot)$ is additively separable, then $Q_i^A < Q_i^*$ for all i .*

If $W(\cdot)$ is affine, the problem of choosing a third-best allocation is equivalent to the problem of choosing public-goods provision levels Q_i^A for $i = 1, \dots, m$, admission prices p_i^A for $i \in J^e$, an expected-payoff function $\bar{v}^A(\cdot)$ and nondecreasing functions $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$ so as to maximize the aggregate per capita surplus

$$\int_0^1 \bar{v}^A(n) dF^n(n) + \sum_{i=1}^m \int_{\hat{\theta}_i^A}^1 (\theta_i Q_i^A - p_i^A) dF^i(\theta_i) \quad (3.35)$$

subject to (3.9) - (3.11), and (3.27). I refer to this problem as the *Generalized Ramsey-Boiteux Problem*. It differs from the Ramsey-Boiteux Problem as discussed in Hellwig (2004 a) by the inclusion of the surplus from production,

$$\int_0^1 \bar{v}^A(n) dF_n(n) = \int_0^1 [\hat{c}^A(n) - \gamma(\hat{y}^A(n), n)] dF^n(n), \quad (3.36)$$

in the objective function and by the inclusion of the revenue from direct taxes,

$$\int_0^1 [\hat{y}^A(n) - \hat{c}^A(n)] dF^n(n) = \int_0^1 T^A(\hat{y}^A(n)) dF^n(n), \quad (3.37)$$

in the government budget constraint.¹⁹

Proposition 3.12 and Corollary 3.13 show that, in the Generalized Ramsey-Boiteux Problem, if $K(\mathbf{Q}^*) > y^*(0) - \gamma(y^*(0), 0)$, it is always desirable to rely on income taxation as an additional source of funds. The optimal marginal income tax is positive at all incomes below the maximum. However, optimal admission fees are also positive, i.e. it is undesirable to rely *only* on income taxes for public-goods finance. The Atkinson-Stiglitz critique of the neglect of direct taxes in the Ramsey-Boiteux approach is thus partly confirmed and partly refuted.

For a given value of the Lagrange multiplier, the first-order conditions (3.31) and (3.32) for optimal public-good provision levels and admission fees in the Generalized Ramsey-Boiteux Problem are identical to the corresponding first-order conditions for the Ramsey-Boiteux problem in Hellwig (2004 a). As discussed there, with $\lambda > 1$, (3.31) implies that the marginal benefits of public good i are less than λ times $\int_0^1 \theta_i dF^i(\theta_i)$, so for an additive cost function, one has underprovision relative to the first-best level Q_i^* . Condition (3.32) can be interpreted as the degenerate form taken by the Ramsey-Boiteux inverse-elasticities formula when variable costs of public-goods admissions are all identically equal to zero.²⁰ The term $(1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A)))$ on the right-hand side indicates the level of aggregate demand for admissions to public good i when the price is p_i^A and the "quality", i.e. the provision level,

¹⁹The Generalized Ramsey-Boiteux Problem would reduce to the Ramsey-Boiteux Problem if I had specified the participation constraint in such a way that private-good production cannot be taxed.

²⁰See, e.g., equation (15-23), p. 467, in Atkinson and Stiglitz (1980). If variable costs are positive, e.g., if costs take the form $K(\mathbf{Q}, U_1, \dots, U_m)$, where, for $i = 1, \dots, m$, $U_i := \int \pi_i dF$ is the aggregate use of public good i , equation (3.32) takes the form

$$(p_i - \frac{\partial K}{\partial U_i}) f_i(\hat{\theta}_i^A) \frac{1}{Q_i} = \frac{\lambda - 1}{\lambda} (1 - F_i(\hat{\theta}_i^A)),$$

is Q_i^A . The term $f^i(\hat{\theta}_i^A) \frac{1}{Q_i^A}$ on the left-hand side indicates the absolute value of the derivative of demand with respect to p_i^A . Condition (3.32) requires admission fees to be chosen in such a way that the elasticities

$$\eta_i^A := \frac{p_i^A}{(1 - F_i(\hat{\theta}_i(p_i^A, Q_i^A)))} f_i(\hat{\theta}_i(p_i^A, Q_i^A)) \frac{1}{Q_i^A} \quad (3.38)$$

of demands for admissions to the different public goods are locally all the same, i.e. that $\eta_i^A = \frac{\lambda-1}{\lambda}$ for all i .

The first-order conditions for optimal admission fees in the Generalized Ramsey-Boiteux Problem also have the same structure as the corresponding conditions (3.15) for the second-best or third-best problem with inequality aversion. By a simple rearrangement of terms, using (3.38), condition (3.15) can be rewritten as

$$\eta_i^A = \frac{\int_{\hat{\theta}_i^A}^1 \int_{[0,1]^m} (\lambda - W') dF(n, \boldsymbol{\theta})}{\lambda(1 - F_i(\hat{\theta}_i(p_i^A, Q_i^A)))}, \quad (3.39)$$

which corresponds to the degenerate form for the case of zero variable costs of the *weighted* inverse-elasticities formula of Diamond-Mirrlees (1971). The weight

$$\frac{\int_{\hat{\theta}_i^A}^1 \int_{[0,1]^m} (\lambda - W') dF(n, \boldsymbol{\theta})}{\lambda(1 - F_i(\hat{\theta}_i(p_i^A, Q_i^A)))} = 1 - \frac{1}{\lambda} \frac{\int_{\hat{\theta}_i^A}^1 \int_{[0,1]^m} W' dF(n, \boldsymbol{\theta})}{\int_{\hat{\theta}_i^A}^1 \int_{[0,1]^m} dF(n, \boldsymbol{\theta})}$$

in (3.39) is a decreasing function of the conditional expectation of the social marginal valuation $W'(v^A(\tilde{n}, \tilde{\boldsymbol{\theta}}))$ of additional consumption for people demanding admission to public good i . The admission fee p_i^A therefore tends to be higher for a public good with a relatively low expected value of $W'(v^A(\tilde{n}, \tilde{\boldsymbol{\theta}}))$, conditional on the information that admission to public good i is requested.

The first-order conditions (3.33) - (3.34) for optimal income taxation in the Generalized Ramsey-Boiteux Problem also have the same structure as the corresponding conditions (3.16) - (3.18) for the second-best problem in Proposition 3.4. The only significant difference between the conditions for

which yields the usual nondegenerate form

$$\frac{p_i - \frac{\partial K}{\partial U_i}}{p_i} = \frac{\lambda - 1}{\lambda} \frac{1}{\eta_i}$$

of the inverse-elasticities formula.

income taxation here and in the second-best problem concerns the positivity of the marginal income tax at the lowest productivity level.

The traditional alignment of nonlinear income taxation with utilitarian redistribution and of public-sector pricing with government budget constraints thus needs to be reconsidered. In the present analysis, the conditions for optimal income taxes and for optimal public-goods admission fees have the same structure, regardless of whether we look at the second-best problem with utilitarian redistribution concerns or at the third-best problem with a government budget constraint based on individual-rationality conditions. Utilitarian concerns for redistribution affect welfare weights, but *not* the overall structure of tax and pricing formulae.

I conclude this section with the observation that the distinction between second-best and third-best allocations is moot if the mechanism designer's inequality aversion is sufficiently large. In this case, the mechanism designer is particularly concerned about the payoffs of people who are worst off in the economy. Second-best allocations then satisfy the participation constraint (3.27) as a matter of course, and one obtains:

Proposition 3.14 *Assume that productivity and taste parameters are affiliated, and let $\{W_k\}$ be any sequence of increasing, concave, and twice continuously differentiable functions on \mathbb{R} such that $\lim_{k \rightarrow \infty} \rho_{W_k}(v) = \infty$, uniformly in v . Then, for any sufficiently large k , second-best and third-best allocations coincide.*

4 Concluding Remarks

I conclude the paper with a few questions for further research. First, in this paper, as in previous work, I have derived the government budget constraint à la Ramsey-Boiteux from a requirement of interim individual rationality for participants. Such a requirement eliminates unrestricted recourse to lump sum taxes, but the resulting government budget constraint is still quite loose and leaves a lot of rooms for cross-subsidization between different activities. Some cross-subsidization may be desirable or even unavoidable, for instance, the use of income taxes or of admission fees on excludable public goods to finance the provision of nonexcludable public goods. However, in practice, a system involving unrestricted cross-subsidization seems undesirable because it provides bad incentives to managers of the different government activities. Such incentive effects have not been part of the analysis here. Developing a tractable model for studying them is an important task for future research. Such a model should be the basis for assessing the costs and benefits of

cross-subsidization between different activities – costs from incentive effects versus benefits from reduced deadweight losses through distortionary taxes and admission fees.²¹

A second set of questions concerns the role of cardinal utility specifications in utilitarian welfare maximization. This paper, like Hellwig (2003), has relied on a particular cardinalization of public-goods preferences. One may feel uneasy about the dependence of the analysis on the cardinalization, but this unease concerns the utilitarian approach as such. Within this approach, the Mirrlees-Seade analysis of optimal income taxation is just as dependent on cardinal properties of utility representations of preferences over consumption-leisure choices as my analysis of optimal utilitarian admission fees. Indeed, within the utilitarian approach, some reliance on a specific cardinalization is unavoidable. However, it would be desirable to have a clear view as to which properties of optimal income taxes and admission fees depend on the ordinal properties of individual utility specifications and which properties depend on the cardinalization.

A final issue concerns the elicitation of preferences as a basis for implementing suitable decisions about the provision of public goods. This issue is a major concern of the theory of public-goods provision. However, in the present paper, in a model of a large economy with independent private values and a law of large numbers, it has not played any role because first-best, second-best and third-best public-goods provision levels are common knowledge. The question is whether this is the price to pay for the drastic simplification of the mechanism design problem that is obtained from the large-economy specification with anonymity. Or is it possible to have large-economy specifications with "aggregate preference shocks", which have the same simple structure for mechanism design and yet leave room for a nontrivial effect of preference elicitation on public-goods provision?

A Appendix: Proofs

A.1 Proofs for Section 2

Lemma 2.1 is practically the same as Lemma 3.1 in Hellwig (2004 a). Therefore the reader is referred to the proof given there.

Proof of Lemma 2.2. By standard arguments, due to Mirrlees (1971,

²¹For a first analysis of such a tradeoff in a somewhat different context, see Ch. 15 of Laffont and Tirole (1993).

1976), incentive compatibility of the allocation A implies that, for any n , the section $v^A(n, \cdot, \dots, \cdot)$ of the expected-payoff function v^A that is determined by n is continuous and convex and has partial derivatives $v_i^A(\cdot)$ satisfying

$$v_i^A(n, \theta_1, \dots, \theta_m) = \chi_i^A(\boldsymbol{\theta}) Q_i^A \quad (\text{A.1})$$

for $i = 1, \dots, m$ and almost all $(\theta_1, \dots, \theta_m) \in [0, 1]^m$. By Lemma 2.1 and (??), renegotiation proofness of A , with associated prices p_1^A, \dots, p_m^A , implies that (2.19) holds for $i = 1, \dots, m$ and almost all $\boldsymbol{\theta} \in [0, 1]^{m+1}$. Convexity of $v^A(n, \cdot, \dots, \cdot)$ and (A.1) imply that, for any i , $\chi_i^A(\boldsymbol{\theta})$ is nondecreasing in θ_i , so (2.19) must actually hold for all (rather than almost all) $\boldsymbol{\theta} \in [0, 1]^{m+1}$. Therefore (A.1) yields

$$v_i^A(n, \theta_1, \dots, \theta_m) = 0 \quad \text{if } \theta_i Q_i^A < p_i^A \quad (\text{A.2})$$

and

$$v_i^A(n, \theta_1, \dots, \theta_m) = Q_i^A \quad \text{if } \theta_i Q_i^A > p_i^A \quad (\text{A.3})$$

for all i and all $\boldsymbol{\theta} = (n, \theta_1, \dots, \theta_m) \in [0, 1]^{m+1}$. By integration, one obtains (2.18) with

$$\bar{v}^A(n) := v^A(n, 0, \dots, 0). \quad (\text{A.4})$$

■

Proof of Proposition 2.3. As mentioned in the text, the argument proceeds in two steps. In the first step, I show that an allocation A is renegotiation proof and incentive compatible if and only if it satisfies statement (a) and the incentive constraint (2.26). The "only if" part of this claim is trivial: By Lemma 2.2, renegotiation proofness and incentive compatibility of A imply the validity of statement (a). The validity of (2.26) follows from the validity of statement (a) and incentive compatibility.

To prove the "if" part of the claim, I note that if the admission indicators $\chi_i^A(n, \boldsymbol{\theta})$ satisfy (2.19), (2.20), then for almost all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, one has $\chi_i^A(n, \boldsymbol{\theta}) = 0$ if $\theta_i Q_i^A < p_i^A$ and $\chi_i^A(n, \boldsymbol{\theta}) = 1$ if $\theta_i Q_i^A > p_i^A$; so renegotiation proofness follows by Lemma 2.1. If statement (a) holds, one also has

$$v(n, \boldsymbol{\theta}) = \bar{v}^A(n) + \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0) \quad (\text{A.5})$$

for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$. From (A.5) and (2.20), one immediately obtains

$$v(n, \boldsymbol{\theta}) \geq \bar{v}^A(n) + \sum_{i=1}^m \bar{\chi}_i^A(\theta'_i) (\theta_i Q_i^A - p_i^A, 0) \quad (\text{A.6})$$

for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$ and all $\boldsymbol{\theta}' = (\theta'_1, \dots, \theta'_m) \in [0, 1]^m$. If (2.26) also holds, it follows that

$$v(n, \boldsymbol{\theta}) \geq \bar{v}^A(n') + \gamma(y^A(n', \boldsymbol{\theta}'), n') - \gamma(y^A(n', \boldsymbol{\theta}'), n) + \sum_{i=1}^m \bar{\pi}_i^A(\theta'_i) (\theta_i Q_i^A - p_i^A, 0) \quad (\text{A.7})$$

for all $(n, \boldsymbol{\theta})$ and all $(n', \boldsymbol{\theta}')$ in $[0, 1]^{m+1}$, so by (2.21) and (2.22), it follows that

$$\begin{aligned} v(n, \boldsymbol{\theta}) &\geq c_0^A(n', \boldsymbol{\theta}') - \gamma(y^A(n', \boldsymbol{\theta}'), n) + \sum_{i=1}^m \bar{\pi}_i^A(\theta'_i) (\theta_i Q_i^A - p_i^A, 0) \\ &= c^A(n', \boldsymbol{\theta}') - \gamma(y^A(n', \boldsymbol{\theta}'), n) + \sum_{i=1}^m \pi_i^A(n', \boldsymbol{\theta}') \theta_i Q_i^A \end{aligned} \quad (\text{A.8})$$

for all $(n, \boldsymbol{\theta})$ and all $(n', \boldsymbol{\theta}')$ in $[0, 1]^{m+1}$, which is just the condition for incentive compatibility of A . The "if" part of the claim is thereby proved, and the first step in the proof of Proposition 2.3 is completed.

For the second step in the proof, I note that, by standard arguments, as in Mirrlees (1976), for any $\boldsymbol{\theta} \in [0, 1]^m$, the incentive compatibility condition (2.26) implies that

$$\bar{v}^A(n) = - \int_0^n \gamma_n(y^A(n', \boldsymbol{\theta}), n') dn' \quad (\text{A.9})$$

for all n . (A.9) implies that the function $n \rightarrow \bar{v}^A(n)$ is absolutely continuous, hence almost everywhere differentiable, with derivative

$$\frac{d\bar{v}^A}{dn}(n) = -\gamma_n(y^A(n, \boldsymbol{\theta}), n). \quad (\text{A.10})$$

Because the left-hand side of (A.10) is independent of $\boldsymbol{\theta}$ and, with $\gamma_{ny} < 0$, the right-hand side is strictly increasing in $y^A(n, \boldsymbol{\theta})$, it follows that $y^A(n, \boldsymbol{\theta})$ is independent of $\boldsymbol{\theta}$; thus there exists a function $\hat{y}^A(\cdot)$ such that $y^A(n, \boldsymbol{\theta}) = \hat{y}^A(n)$ for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, so (2.26) becomes (2.25). ■

As mentioned in the text, Remark 2.4 follows from the arguments of Mirrlees (1976), pp. 334 f., and is not again proved here.

Proof of Remark 2.5. The first statement of remark is equivalent to the statement that the map

$$n \rightarrow \hat{y}^A(n) - \bar{v}^A(n) - \gamma(\hat{y}^A(n), n) \quad (\text{A.11})$$

from $[0, 1]$ into \mathfrak{R} is measurable with respect to the σ -algebra on $[0, 1]$ that is generated by the map $n \rightarrow \hat{y}^A(n)$, i.e. that

$$\hat{y}^A(n) - \bar{v}^A(n) - \gamma(\hat{y}^A(n), n) = \hat{y}^A(\hat{n}) - \bar{v}^A(\hat{n}) - \gamma(\hat{y}^A(\hat{n}), \hat{n}) \quad (\text{A.12})$$

whenever $\hat{y}^A(n) = \hat{y}^A(\hat{n})$. To establish this property, suppose that $\hat{y}^A(n) = \hat{y}^A(\hat{n})$ for some n, \hat{n} , and, without loss of generality, let $\hat{n} < n$. By Proposition 2.3 and Remark 2.4, the function $\hat{y}^A(\cdot)$ is nondecreasing, and one has $\hat{y}^A(n') = \hat{y}^A(n)$ for all $n' \in [\hat{n}, n]$. From (2.28) and (2.32), one then has

$$\bar{v}^A(n) = \bar{v}^A(\hat{n}) - \int_{\hat{n}}^n \gamma_n(\hat{y}^A(n), n') dn',$$

hence

$$\bar{v}^A(n) = \bar{v}^A(\hat{n}) - \gamma(\hat{y}^A(n), n) + \gamma(\hat{y}^A(n), \hat{n})$$

and

$$\hat{y}^A(n) - \bar{v}^A(n) - \gamma(\hat{y}^A(n), n) = \hat{y}^A(n) - \bar{v}^A(\hat{n}) - \gamma(\hat{y}^A(n), \hat{n}).$$

Because $\hat{y}^A(n) = \hat{y}^A(\hat{n})$, (A.12) follows immediately, and the first statement of the remark is proved.

Using (2.28), one can rewrite (2.35) in the form

$$\begin{aligned} T^A(\hat{y}^A(n)) &= \hat{y}^A(n) - \bar{v}^A(0) - \gamma(\hat{y}^A(n), n) + \int_0^n \gamma_n(\hat{y}^A(n'), n') dn' \\ &= \hat{y}^A(0) - \bar{v}^A(0) - \gamma(\hat{y}^A(0), 0) + \int_0^n (1 - \gamma_y(\hat{y}^A(n'), n')) d\hat{y}^A(n'). \end{aligned}$$

The second statement of the remark follows immediately. ■

A.2 Proofs for Section 3

Proposition 3.1 is standard, so its proof is left to the reader. Corollary 3.2 follows from Proposition 3.1 and the argument given in the text.

Proof of Corollary 3.3. Given that a first-best allocation is renegotiation proof, by Proposition 2.3 and Remark 2.4 the allocation is incentive compatible if and only if

$$c^A(n, \theta) = \bar{v}^A(0) - \int_0^n \gamma_n(y^*(n'), n') dn' + \gamma(y^*(n), n) \quad (\text{A.13})$$

for all n and θ . By (3.1), (A.13) can be rewritten as

$$\begin{aligned} c^A(n, \theta) &= \bar{v}^A(0) - \int_0^n \frac{d}{dn} (y^*(n') - \gamma(y^*(n'), n')) dn' + \gamma(y^*(n), n) \\ &= \bar{v}^A(0) + y^*(n) - [y^*(0) - \gamma(y^*(0), 0)]. \end{aligned} \quad (\text{A.14})$$

Given (A.14), (3.7) follows from the feasibility condition (2.31). ■

Proof of Proposition 3.4. Given the allocation A , define a function W_0^A by setting

$$W_0^A(v, n, \mathbf{Q}^A, \mathbf{p}^A) := \int_{[0,1]^m} W \left(v + \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0) \right) dF(\theta|n) \quad (\text{A.15})$$

for any $v \in \mathfrak{R}$, $n \in [0, 1]$, $\mathbf{Q}^A \in \mathfrak{R}_+^m$, and $\mathbf{p}^A \in \mathfrak{R}_+^m$. If A is a second-best allocation, then $\mathbf{Q}^A, \mathbf{p}^A$, and the associated consumption, output provision and payoff functions $\hat{c}^A(\cdot), \hat{y}^A(\cdot), \bar{v}^A(\cdot)$ must also be a solution to the problem of maximizing

$$\int_{[0,1]} W_0(\bar{v}^A(n), n, \mathbf{Q}^A, \mathbf{p}^A) f^n(n) dn \quad (\text{A.16})$$

subject to (3.9) - (3.11) and the requirement that $\hat{y}^A(\cdot)$ be nondecreasing. This problem in turn is equivalent to the problem of choosing $\mathbf{Q}^A, \mathbf{p}^A, \hat{y}^A(\cdot)$ and $\bar{v}^A(\cdot)$ so as to maximize (A.16) subject to the feasibility constraint

$$\int_0^1 [\hat{y}^A(n) - \bar{v}^A(n) - \gamma(\hat{y}^A(n), n)] f^n(n) dn \leq K(Q_1^A, \dots, Q_m^A) - \sum_{i=1}^m p_i^A (1 - F^i(\hat{\theta}_i(p_i^A, Q_i^A))), \quad (\text{A.17})$$

the incentive compatibility condition

$$\frac{d\bar{v}^A}{dn} = -\gamma_n(\hat{y}^A(n), n) \quad (\text{A.18})$$

and the monotonicity requirement on $\hat{y}^A(\cdot)$. Except for the fact that the integrand in (A.16) depends on $\mathbf{Q}^A, \mathbf{p}^A$ and the productivity parameter n as well as the payoff level $\bar{v}^A(n)$, the latter problem is a standard problem of optimal utilitarian income taxation à la Mirrlees (1971, 1976) or Seade (1977, 1982) and can be handled by control theoretic methods. As discussed in Hellwig (2004 c), for any solution $\mathbf{Q}^A, \mathbf{p}^A, \hat{y}^A(\cdot), \bar{v}^A(\cdot)$ to this problem, there exists a real number $\lambda \geq 0$ and there exist two absolutely continuous

functions $\psi^A(\cdot), \chi^A(\cdot)$ such that (3.13) holds for $i \in J^{ne}$, (3.14) and (3.15) hold for $i \in J^e$, and, for any n , one has

$$\frac{d\psi^A}{dn} = - \left[\frac{\partial W_0^A}{\partial v}(\bar{v}^A(n), n, \mathbf{Q}^A, \mathbf{p}^A) - \lambda \right] f^n(n), \quad (\text{A.19})$$

$$\frac{d\chi^A}{dn} = -\lambda(1 - \gamma_y(\hat{y}^A(n), n))f^n(n) + \psi^A(n)\gamma_{ny}(\hat{y}^A(n), n), \quad (\text{A.20})$$

and

$$\chi^A(n) \leq 0, \quad (\text{A.21})$$

the latter condition holding with equality unless $\hat{y}^A(n') = \hat{y}^A(n)$ for all n' in some open neighbourhood of n . Moreover, $\psi^A(\cdot)$ and $\chi^A(\cdot)$ satisfy the transversality conditions

$$\psi^A(0) = \psi^A(1) = 0 \quad (\text{A.22})$$

and

$$\chi^A(1) = 0. \quad (\text{A.23})$$

By a straightforward integration, (A.19) and (A.22) yield

$$\lambda = \int_0^1 \frac{\partial W_0^A}{\partial v}(\bar{v}^A(n), n, \mathbf{Q}^A, \mathbf{p}^A) f^n(n) dn, \quad (\text{A.24})$$

which implies (3.12). By another integration, (A.20) and (A.23) yield

$$\chi^A(n) + \int_n^1 [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n'))f^n(n') + \psi^A(n')\gamma_{ny}(\hat{y}^A(n'), n')] = 0, \quad (\text{A.25})$$

for all n , so (A.21) implies that

$$\int_n^1 [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n'))f^n(n') + \psi^A(n')\gamma_{ny}(\hat{y}^A(n'), n')] \geq 0, \quad (\text{A.26})$$

the inequality holding as an equation unless $\hat{y}^A(n') = \hat{y}^A(n)$ for all n' in some open neighbourhood of n . If $\hat{y}^A(\cdot)$ is strictly increasing on some open neighbourhood of n , one must have $\chi^A(n') = 0$ and $\frac{d\chi^A}{dn}(n') = 0$ for all n' belonging to this open neighbourhood, hence in particular,

$$-\lambda(1 - \gamma_y(\hat{y}^A(n), n))f^n(n) + \psi^A(n)\gamma_{ny}(\hat{y}^A(n), n) = 0. \quad (\text{A.27})$$

■

Proof of Proposition 3.5. I first show that, under the given assumptions, for $v^A(n, \boldsymbol{\theta})$ satisfying

$$v^A(n, \boldsymbol{\theta}) = \bar{v}^A(n) + \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0), \quad (\text{A.28})$$

for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, the map

$$n' \rightarrow \int_{[0,1]^m} W'(v^A(n', \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n') \quad (\text{A.29})$$

is strictly decreasing. Let n_1 and n_2 be such that $n_1 < n_2$, and note that, by the concavity of W , the function $(n', \boldsymbol{\theta}) \rightarrow W'(v^A(n_2, \boldsymbol{\theta}))$ is nonincreasing. By Theorem 5, p. 1100, of Milgrom and Weber (1982) therefore, the affiliation assumption on F implies that

$$\int_{[0,1]^m} W'(v^A(n_2, \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n_1) \geq \int_{[0,1]^m} W'(v^A(n_2, \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n_2).$$

By the strict concavity of W and the strict monotonicity of $\bar{v}^A(\cdot)$, it follows that

$$\int_{[0,1]^m} W'(v^A(n_1, \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n_1) > \int_{[0,1]^m} W'(v^A(n_2, \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n_2),$$

as claimed.

Given the strict monotonicity of (A.29), there exists a unique \hat{n} such that

$$\int_{[0,1]^m} W'(v^A(n', \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n') \leq \lambda \quad \text{as } n' \geq \hat{n}. \quad (\text{A.30})$$

From (A.30), one infers that the function

$$n \rightarrow \psi^A(n) = \int_n^1 \left[\int_{[0,1]^m} W'(v^A(n', \boldsymbol{\theta})) dF(\boldsymbol{\theta}|n') - \lambda \right] dF^n(n')$$

is decreasing for $n < \hat{n}$ and increasing for $n > \hat{n}$. Because (A.22) implies $\psi^A(1) = \psi^A(0) = 0$, it follows that $\hat{n} \in (0, 1)$ and that $\psi^A(n) < 0$ for all $n \in (0, 1)$.

From here on, the argument is routine: If $n \in (0, 1)$ is such that $\hat{y}^A(\cdot)$ is strictly increasing at n , Proposition 3.4 implies

$$\int_n^1 [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n')) f^n(n') + \gamma_{ny}(\hat{y}^A(n'), n') \psi^A(n')] dn' = 0$$

as well as

$$\int_{\hat{n}}^1 [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n'))f^n(n') + \gamma_{ny}(\hat{y}^A(n'), n')\psi^A(n')] dn' \geq 0$$

for all $\hat{n} > n$, hence

$$\int_n^{\hat{n}} [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n'))f^n(n') + \gamma_{ny}(\hat{y}^A(n'), n')\psi^A(n')] dn' \leq 0$$

for all $\hat{n} > n$. It follows that there exists a sequence $\{n^k\}$ converging to n from above such that

$$-\lambda(1 - \gamma_y(\hat{y}^A(n^k), n^k))f^n(n^k) + \gamma_{ny}(\hat{y}^A(n^k), n^k)\psi^A(n^k) \leq 0 \quad (\text{A.31})$$

for all k . Exploiting the monotonicity of $\hat{y}^A(\cdot)$, one may define $\bar{y}^A(n) = \lim_{n^k \downarrow n} \hat{y}^A(n^k)$. Upon taking limits in (A.31), relying on the continuity of γ_y , γ_{ny} , and ψ^A , one obtains $-\lambda(1 - \gamma_y(\bar{y}^A(n), n))f^n(n) + \gamma_{ny}(\bar{y}^A(n), n)\psi^A(n) \leq 0$, hence $\gamma_y(\bar{y}^A(n), n) < 1$. By the monotonicity of $\hat{y}^A(\cdot)$, one also has $\hat{y}^A(n) \leq \bar{y}^A(n)$, so by the convexity of $\gamma(\cdot, n)$, $\gamma_y(\hat{y}^A(n), n) < 1$ implies $\gamma_y(\hat{y}^A(n), n) < 1$.

Alternatively, if $\hat{y}^A(\cdot)$ is constant on some open neighbourhood of n , there exists $\hat{n} < n$ such that $\hat{y}^A(\cdot)$ is strictly increasing at \hat{n} , and moreover, $\hat{y}^A(n') = \hat{y}^A(n)$ for all $n' \in (\hat{n}, n]$. By the argument just given, one has $\lim_{n' \downarrow \hat{n}} \gamma_y(\hat{y}^A(n'), n') < 1$, hence $\gamma_y(\hat{y}^A(\hat{n}), \hat{n}) < 1$. Because $\gamma_{ny} < 0$, it follows that $\gamma_y(\hat{y}^A(n), n) < 1$.

Turning to the behaviour of $\hat{y}^A(\cdot)$ for n close to one, I note that (3.16) can be rewritten in the form

$$\frac{\int_n^1 \gamma_{ny}(\hat{y}^A(n'), n')\psi^A(n')dn'}{1 - n} \geq \frac{\int_n^1 \lambda(1 - \gamma_y(\hat{y}^A(n'), n'))f^n(n')dn'}{1 - n}. \quad (\text{A.32})$$

Because $\gamma_y(\hat{y}^A(n'), n') < 1$ for $n' \in (0, 1)$, the right-hand side of (A.32) is nonnegative. Because $\psi^A(1) = 0$, the left-hand side of (A.32) converges to zero as n converges to one. The right-hand side of (A.32) must then also converge to zero. Given the monotonicity of $\hat{y}^A(\cdot)$, it follows that $\lim_{n' \uparrow 1} \gamma_y(\hat{y}^A(n'), n') = 1$.

As for the behaviour of $\hat{y}^A(\cdot)$ for n close to zero, if there exists a sequence $\{n^k\}$ of points at which $\hat{y}^A(\cdot)$ is increasing, then by Proposition 3.4, one has

$$\int_{n^{k+1}}^{n^k} [-\lambda(1 - \gamma_y(\hat{y}^A(n'), n'))f^n(n') + \gamma_{ny}(\hat{y}^A(n'), n')\psi^A(n')] dn' = 0$$

for all k . Then there exist sequences $\{\hat{n}^k\}, \{\bar{n}^k\}$ converging to zero such that for all k one has

$$-\lambda(1 - \gamma_y(\hat{y}^A(\hat{n}^k), \hat{n}^k))f^n(\hat{n}^k) + \gamma_{ny}(\hat{y}^A(\hat{n}^k), \hat{n}^k)\psi^A(\hat{n}^k) \leq 0$$

and

$$-\lambda(1 - \gamma_y(\bar{y}^A(\bar{n}^k), \bar{n}^k))f^n(\bar{n}^k) + \gamma_{ny}(\bar{y}^A(\bar{n}^k), \bar{n}^k)\psi^A(\bar{n}^k) \geq 0.$$

Upon defining $\bar{y}^A(0) := \lim_{n \downarrow 0} \hat{y}^A(n)$ and taking limits, as before, one obtains

$$-\lambda(1 - \gamma_y(\bar{y}^A(0), 0))f^n(0) + \gamma_{ny}(\bar{y}^A(0), 0)\psi^A(0) \leq 0$$

and

$$-\lambda(1 - \gamma_y(\bar{y}^A(0), 0))f^n(0) + \gamma_{ny}(\bar{y}^A(0), 0)\psi^A(0) \geq 0,$$

hence $\lambda(1 - \gamma_y(\bar{y}^A(0), 0))f^n(0) = \gamma_{ny}(\bar{y}^A(0), 0)\psi^A(0)$. Because $\psi^A(0) = 0$, it follows that $\gamma_y(\bar{y}^A(0), 0) = 1$ and hence that $\lim_{n \downarrow 0} \gamma_y(\hat{y}^A(n), n) = 1$. ■

Corollary 3.6 follows immediately from Proposition 3.5 and Remark 2.5. Proposition 3.7 follows from Proposition 3.4 and the arguments given in the text.

The proof of Proposition 3.8 relies on the following lemma.

Lemma A.1 *There exists $B > 0$, such that*

$$0 \leq v^A(n, \theta) - v^A(0, \mathbf{0}) \leq B \quad (\text{A.33})$$

for all $(n, \theta) \in [0, 1]^{m+1}$ and any allocation A which is second-best for some welfare function W .

Proof. Let A be a second-best allocation for some welfare function W . By Lemma 2.2, (A.33) is equivalent to

$$0 \leq \bar{v}^A(n) - \bar{v}^A(0) + \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0) \leq B. \quad (\text{A.34})$$

From (2.28), one has

$$0 \leq \bar{v}^A(n) - \bar{v}^A(0) \leq \bar{v}^A(1) - \bar{v}^A(0) \leq \gamma(\hat{y}^A(1), 0) - \gamma(\hat{y}^A(1), 1)$$

for all $n \in [0, 1]$. From Proposition 3.5, one also has $\hat{y}^A(1) = y^*(1)$, where $y^*(1)$ is again the first-best output provision level for the productivity parameter $n = 1$. Hence

$$0 \leq \bar{v}^A(n) - \bar{v}^A(0) \leq \gamma(y^*(1), 0) - \gamma(y^*(1), 1) \quad (\text{A.35})$$

for all $n \in [0, 1]$. Turning to the other term in (A.34), one obviously has

$$0 \leq \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^{A^k}, 0) \leq \sum_{i=1}^m Q_i^A.$$

By Proposition 3.7, one has $K_i(\mathbf{Q}^A) \in [0, \int_0^1 \theta_i dF_i(\theta_i)]$ for $i = 1, \dots, m$. By the strict convexity of $K(\cdot)$, in combination with the continuity and boundary conditions on $K_i(\cdot)$, $i = 1, \dots, m$, it follows that \mathbf{Q}^A belongs to a compact set $\mathcal{Q} \subset \mathfrak{R}_+^m$, which is independent of A and W . Upon choosing \bar{Q} so that $\sum_{i=1}^m Q_i$ for all $\mathbf{Q} \in \mathcal{Q}$, one concludes that

$$0 \leq \sum_{i=1}^m \max(\theta_i Q_i^A - p_i^A, 0) \leq \bar{Q} \quad (\text{A.36})$$

for all $\boldsymbol{\theta} \in [0, 1]^n$. If one sets $B = \gamma(y^*(1), 0) - \gamma(y^*(1), 1) + \bar{Q}$, one finds that (A.34), and hence (A.33), hold for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$. ■

Proof of Proposition 3.8. (a) Let $R > 0$ be such that

$$\sup_{\hat{\theta}_i \in [0, 1]} \frac{F_i(\hat{\theta}_i)}{F_i(\hat{\theta}_i) + \hat{\theta}_i f_i(\hat{\theta}_i)} < e^{-RB} \quad (\text{A.37})$$

for all i , where B is given by Lemma A.1. Let the welfare function W be such that $\rho_W(v) \leq R$ for all v , and let A be a second-best allocation for W . By a straightforward integration, one obtains

$$W'(v^A(n, \boldsymbol{\theta})) \geq W'(v^A(0, \mathbf{0})) \exp(-R[v^A(n, \boldsymbol{\theta}) - v^A(0, \mathbf{0})]),$$

so (A.34) implies that

$$W'(v^A(n, \boldsymbol{\theta})) \leq W'(v^A(0, \mathbf{0})) e^{-RB} \quad (\text{A.38})$$

for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$. Upon combining (A.37) and (A.38), one finds that

$$-(F_i(\hat{\theta}_i) + \hat{\theta}_i f_i(\hat{\theta}_i)) W'(v^A(n, \boldsymbol{\theta})) + W'(v^A(0, \mathbf{0})) \int_0^{\hat{\theta}_i} dF_i(\theta'_i) < 0$$

for all $(n, \boldsymbol{\theta}) \in [0, 1]^{m+1}$, all i and all $\hat{\theta}_i \in (0, 1]$. Because $W'(\cdot)$ is nondecreasing, it follows that

$$-(F_i(\hat{\theta}_i) + \hat{\theta}_i f_i(\hat{\theta}_i)) \lambda + \int_0^{\hat{\theta}_i} \int_{[0, 1]^m} W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) < 0,$$

for all i and all $\hat{\theta}_i \in (0, 1]$, which is equivalent to

$$(1 - F_i(\hat{\theta}_i) - \hat{\theta}_i f_i(\hat{\theta}_i))\lambda - \int_{\hat{\theta}_i}^1 \int_{[0,1]^m} W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) < 0$$

for all i and all $\hat{\theta}_i \in (0, 1]$. For any i , therefore, $\hat{\theta}_i^A = 0$ is the unique solution to the first-order condition (3.15), and one must have $p_i^A = 0$. The first statement in part (a) of the proposition is thereby proved. The second statement in (a) follows from a standard upper hemi-continuity argument and Corollary 3.3.

(b) Let v^* be the maximum of $v^A(0, \mathbf{0})$ over the set of allocations A that are feasible, incentive compatible and renegotiation proof, and let A^* be an allocation at which the maximum is reached, so $v^{A^*}(0, \mathbf{0}) = v^*$. I claim that for any allocation A satisfying (A.33), as well as $v^A(0, \mathbf{0}) < v^*$, one has

$$\int_{[0,1]^{m+1}} W(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) < \int_{[0,1]^{m+1}} W(v^{A^*}(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) \quad (\text{A.39})$$

for any welfare function W such that $\rho_W(v)$ is sufficiently large, uniformly in v . To prove this claim, I note that, for any concave function W , any v and any $\delta \geq 0$, one has

$$W(v) \leq W(v^* + \delta) + W'(v^* + \delta)(v - v^* - \delta); \quad (\text{A.40})$$

in particular,

$$W(v) \leq W(v^*) + W'(v^*)(v - v^*). \quad (\text{A.41})$$

For any allocation A , let G^A be the distribution of the payoff $v^A(\tilde{n}, \tilde{\boldsymbol{\theta}})$ that is induced by A , so one has

$$\int_{[0,1]^{m+1}} W(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) = \int_{\mathfrak{R}} W(v) dG^A(v).$$

Upon applying (A.41) for $v \leq v^*$ and (A.40) for $v > v^*$, one finds that

$$\begin{aligned} \int_{\mathfrak{R}} W(v) dG^A(v) &\leq \int_{-\infty}^{v^*} [W(v^*) + W'(v^*)(v - v^*)] dG^A(v) \\ &\quad + \int_{v^*}^{\infty} [W(v^* + \delta) + W'(v^* + \delta)(v - v^* - \delta)] dG^A(v) \end{aligned}$$

for all $\delta \geq 0$. If the allocation A satisfies (A.33), it follows that

$$\begin{aligned} \int_{\Re} W(v) dG^A(v) &\leq W(v^*) + W'(v^*) \int_{-\infty}^{v^*} (v - v^*) dG^A(v) \\ &\quad + W(v^* + \delta) - W(v^*) + W'(v^* + \delta)(B - v^* - \delta) \int_{v^*}^{\infty} dG^A(v) \\ &\leq W(v^*) + W'(v^*) \int_{-\infty}^{v^*} (v - v^*) dG^A(v) \\ &\quad + W'(v^*)\delta + W'(v^* + \delta)(B - v^*) \end{aligned}$$

for all $\delta \geq 0$. If W is such that $\rho_W(v) \geq \bar{R}$ for all v , it follows that

$$\begin{aligned} \int_{\Re} W(v) dG^A(v) &\leq W(v^*) + W'(v^*) \int_{-\infty}^{v^*} (v - v^*) dG^A(v) \quad (\text{A.42}) \\ &\quad + W'(v^*)[\delta + e^{-\bar{R}\delta}(B - v^*)] \end{aligned}$$

for all $\delta \geq 0$. If A is such that $G^A((-\infty, v^*)) > 0$, then for \bar{R} sufficiently large and, e.g., $\delta = \bar{R}^{-1/2}$, (A.42) implies

$$\int_{\Re} W(v) dG^A(v) < W(v^*),$$

and (A.39) follows immediately.

For any sequence $\{W_k\}$ of increasing, concave and twice continuously differentiable functions on \Re such that $\lim_{k \rightarrow \infty} \rho_{W_k}(v) = \infty$, uniformly in v , an associated sequence $\{A^k\}$ of second-best allocations must therefore satisfy $\lim_{k \rightarrow \infty} G^{A^k}((-\infty, v^*)) = 0$, or

$$\lim_{k \rightarrow \infty} F(\{(n, \theta) | v^{A^k}(n, \theta) < v^*\}) = 0. \quad (\text{A.43})$$

One easily verifies that (A.43) is only possible if $\lim_{k \rightarrow \infty} v^{A^k}(0, \mathbf{0}) = v^*$. By continuity, it follows that any limit point A^∞ of the sequence $\{A^k\}$ satisfies $v^{A^\infty}(0, \mathbf{0}) = v^*$ and is therefore a maximizer of $v^A(0, \mathbf{0})$ over the set of feasible, renegotiation proof and incentive-compatible allocations. Part (b) of the proposition follows immediately. ■

Proof of Proposition 3.10. Proceeding exactly as in the proof of Proposition 3.4, one finds that for any third-best allocation A with associated entry fees p_i^A , $i \in J^e$, and consumption, output provision and payoff functions $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$, there exists a real number $\lambda \geq 0$ and there exist two absolutely continuous functions $\psi^A(\cdot)$, $\chi^A(\cdot)$ such that, (3.13) holds

for $i \in J^{ne}$, (3.14) and (3.15) hold for $i \in J^e$, and for any n , (A.19) - (A.21) and (A.23) are satisfied, where (A.21) again holds with equality unless $\hat{y}^A(n') = \hat{y}^A(n)$ for all n' in some open neighbourhood of n . However, the transversality conditions for $\psi^A(\cdot)$ are now

$$\psi^A(1) = 0 \quad (\text{A.44})$$

and

$$\psi^A(0) \leq 0, \quad (\text{A.45})$$

with the complementary-slackness condition

$$\psi^A(0)\bar{v}^A(0) = 0. \quad (\text{A.46})$$

From (A.19) and (A.44), one obtains

$$\psi^A(0) = \int_0^1 \frac{\partial W_0^A}{\partial v}(\bar{v}^A(n), n, \mathbf{Q}^A, \mathbf{p}^A) f^n(n) dn - \lambda. \quad (\text{A.47})$$

Upon using (A.47) to substitute for $\psi^A(0)$ in (A.45) and (A.46), one obtains (3.29) as well as the complementary-slackness condition

$$\left[\int_0^1 W'(v^A(n, \boldsymbol{\theta})) dF(n, \boldsymbol{\theta}) - \lambda \right] \bar{v}^A(0) = 0.$$

■

Proof of Proposition 3.11. If the inequality in (3.29) is strict, there exists $\hat{n} > 0$ such that the costate variable $\psi^A(\cdot)$ satisfies $\psi^A(n) < 0$ for all $n \in [0, \hat{n})$. The first statement of the proposition then follows by the same argument as in the proof of Proposition 3.5. One notes that, for any $i \in J^e$, at $\hat{\theta}_i^A = 0$, the left-hand side of (3.15) is positive if the inequality in (3.29) is strict. The first-order condition (3.15) is thus incompatible with $p_i^A = 0$.

■

Proof of Proposition 3.12. If $W(\cdot)$ is affine, there is no loss of generality in assuming that $W'(v) = 1$ for all v . If the allocation A , with associated p_1^A, \dots, p_m^A and $\hat{c}^A(\cdot)$, $\hat{y}^A(\cdot)$, $\bar{v}^A(\cdot)$, is a solution to the third-best welfare maximization problem when $W'(\cdot) \equiv 1$, then by Proposition 3.10, there exists $\lambda \geq 1$ such that (3.31) - (3.34) hold. To complete the proof, it suffices to show that $\lambda > 1$. If λ were equal to one, one should have $\mathbf{Q}^A = \mathbf{Q}^*$, $p_i^A = 0$ for all i and

$$\int_n^1 (1 - \gamma_y(\hat{y}^A(n'), n')) f^n(n') dn' \leq 0 \quad (\text{A.48})$$

for all $n \in [0, 1]$. Moreover, if the inequality in (A.48) were strict, then $\hat{y}^A(\cdot)$ would be constant on some open neighbourhood of n . If $\hat{y}^A(\cdot)$ were strictly increasing at n , the inequality in (A.48) would hold as an equation. However, the same arguments as in the proof of Proposition 3.5 imply that $\gamma_y(\hat{y}^A(n), n) \leq 1$ for all n , so (A.48) is only possible if one has $\gamma_y(\hat{y}^A(n), n) = 1$ and $\hat{y}^A(n) = y^*(n)$ for all n . By Remark 2.5, it follows that $\hat{y}^A(n) - \hat{c}^A(n) = \hat{y}^A(0) - \hat{c}^A(0)$ for all n . Given that $\mathbf{Q}^A = \mathbf{Q}^*$, $p_i^A = 0$ for all i and $\hat{y}^A(n) - \hat{c}^A(n) = \hat{y}^A(0) - \hat{c}^A(0)$ for all n , the feasibility constraint implies $\hat{c}^A(0) \leq \hat{y}^A(0) - K(\mathbf{Q}^*)$. By Proposition 2.3 and the participation constraint, it follows that $0 \leq \bar{v}^A(0) \leq \hat{y}^A(0) - K(\mathbf{Q}^*) - \gamma(\hat{y}^A(0), 0)$ and hence that $K(\mathbf{Q}^*) \leq y^*(0) - \gamma(y^*(0), 0)$. The assumption that one can have $\lambda = 1$ when $K(\mathbf{Q}^*) > y^*(0) - \gamma(y^*(0), 0)$ has thus led to a contradiction and must be false. Hence $K(\mathbf{Q}^*) > y^*(0) - \gamma(y^*(0), 0)$ implies $\lambda > 1$. ■

Proof of Proposition 3.14. Let $\{A^k\}$ be a sequence of second-best allocations associated with the sequence $\{W_k\}$. By Proposition 3.4, the feasibility constraint (3.9) is always binding, so one has

$$\int_0^1 [\hat{y}^{A^k}(n) - \hat{c}^{A^k}(n)] dF^n(n) = K(Q_1^{A^k}, \dots, Q_m^{A^k}) - \sum_{i=1}^m p_i^{A^k} (1 - F^i(\hat{\theta}_i(p_i^{A^k}, Q_i^{A^k})))$$

for all k . By Remark 2.5, Corollary 3.6, and Proposition 3.8, it follows that

$$T^{A^k}(0) = \hat{y}^{A^k}(0) - \hat{c}^{A^k}(0) < 0,$$

and therefore

$$\bar{v}^{A^k}(0) > \hat{y}^{A^k}(0) - \gamma(\hat{y}^{A^k}(0), 0)$$

for any sufficiently large k . By Proposition 3.5, one also has $\hat{y}^{A^k}(0) \in [0, y^*(0)]$ for all k . Since $\gamma(0, 0) = 0$, It follows that $\hat{y}^{A^k}(0) \geq \gamma(\hat{y}^{A^k}(0), 0)$ for all k and, hence, that $\bar{v}^{A^k}(0) > 0$ for any sufficiently large k . For any sufficiently large k , the second-best allocation A^k thus satisfies the participation constraint; consequently, it is also a third-best allocation. Any other third-best allocation must then provide the same overall welfare as the second-best allocation A^k and must also be second-best. ■

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